

Advanced composites in engineering structures

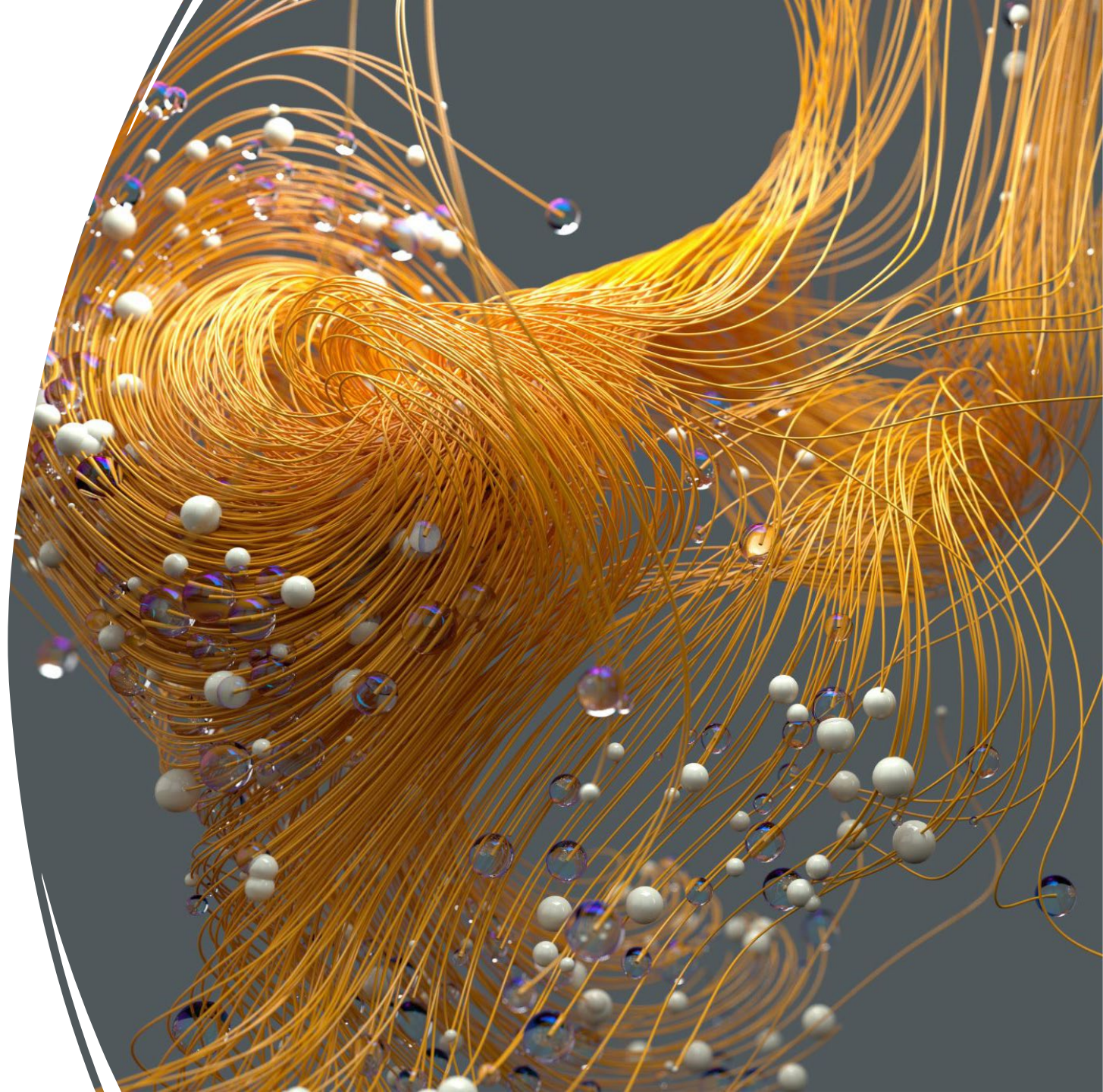
Anastasios P. Vassilopoulos

Lecture #5

Strength of composite materials

Composite failure

- **Composites** fail in a different way compared to metals
- Exhibit different failure modes
- Brittle fibers in a ductile matrix
 - Sudden failure
 - Crazeing and matrix cracking might occur
 - Delamination and debonding



Metals

Crack length

Composites

Broken fibers

Delamination

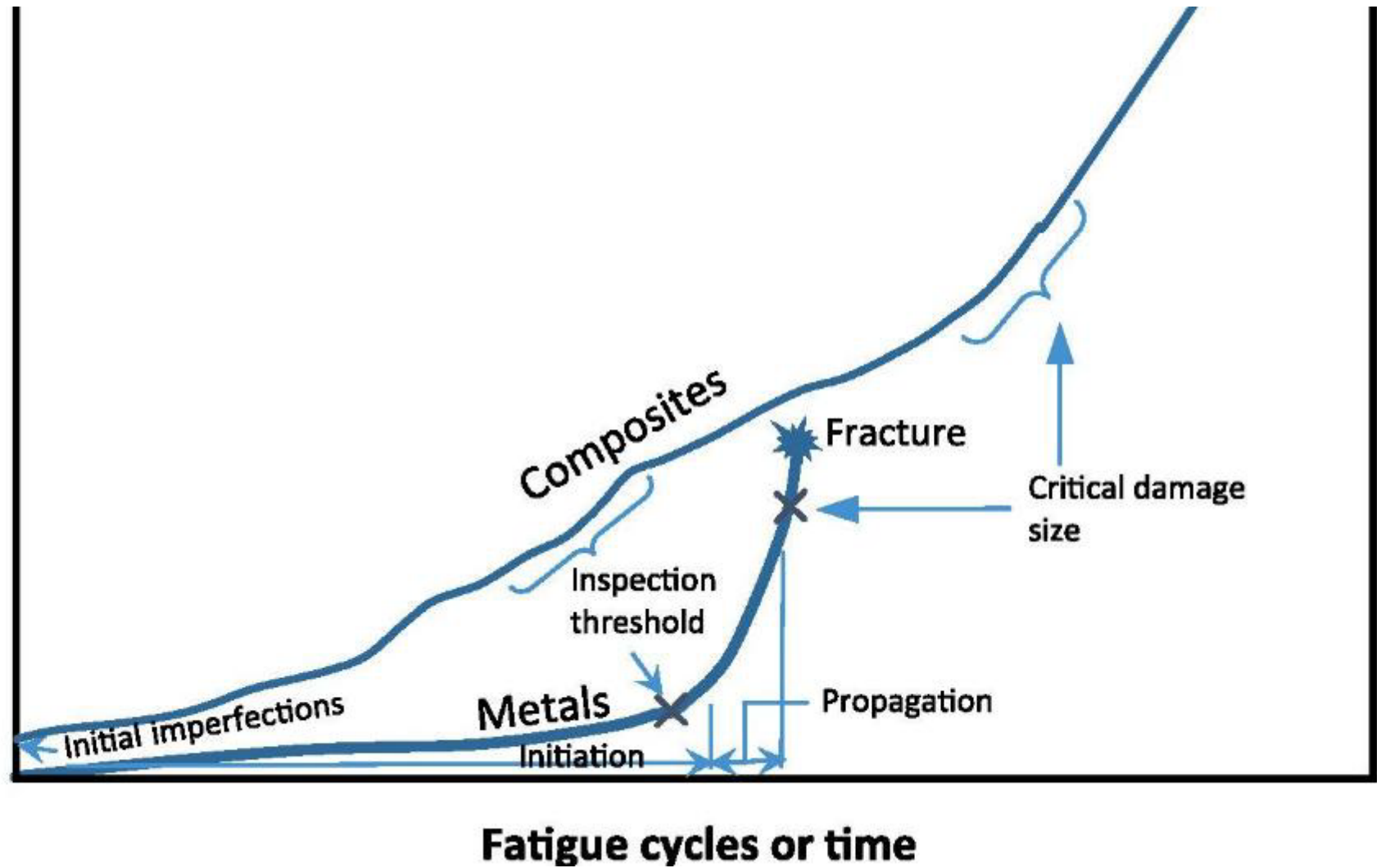
Matrix cracking

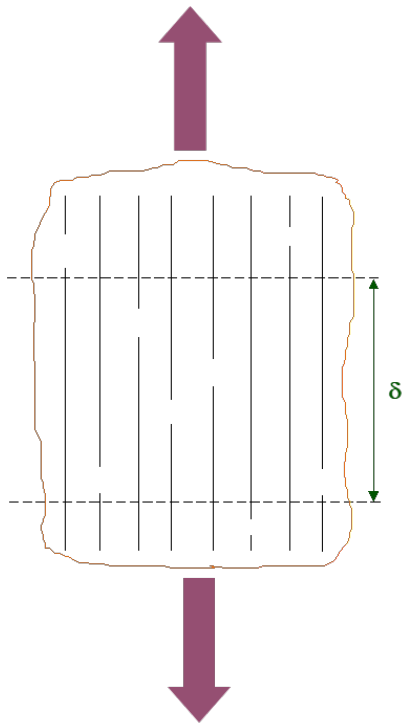
Composite

cracking

Debonding

Voids

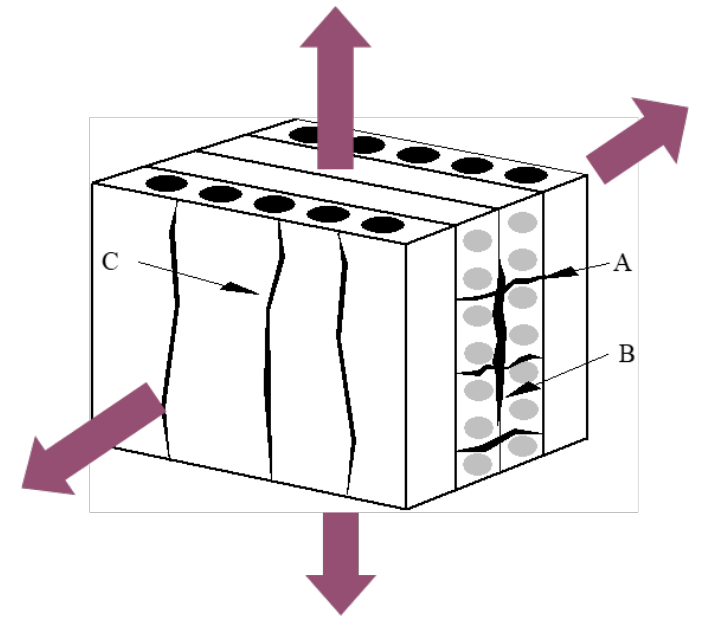




Tensile Failure

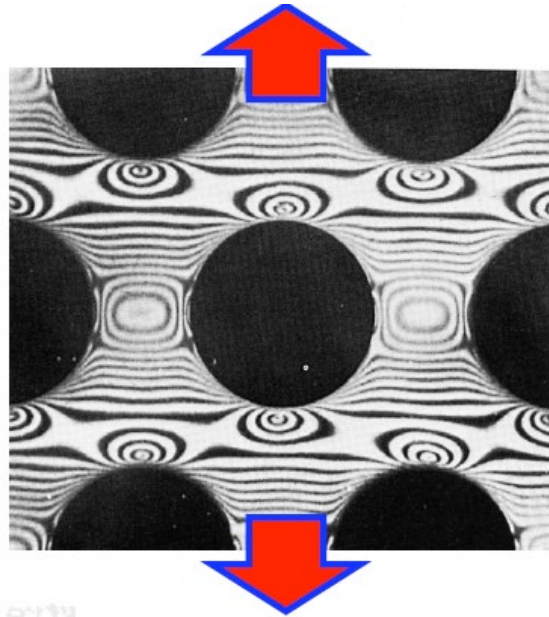


Compression Failure

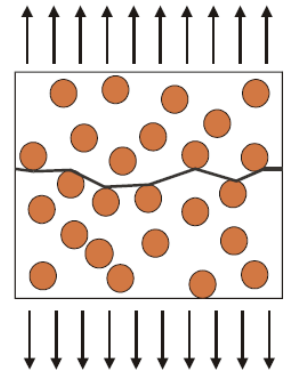


Matrix Cracking
& Delamination

Damage Modes

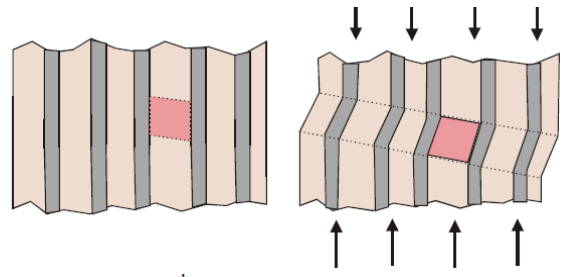


Transverse tension

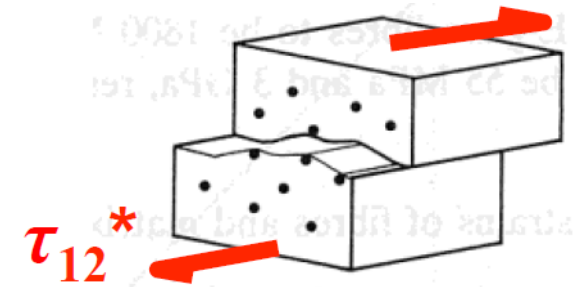


Damage Modes

Axial compression

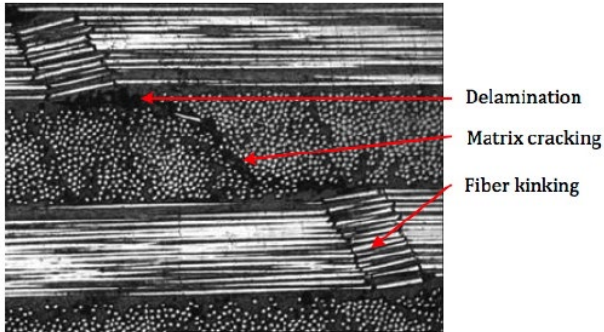


Axial shear

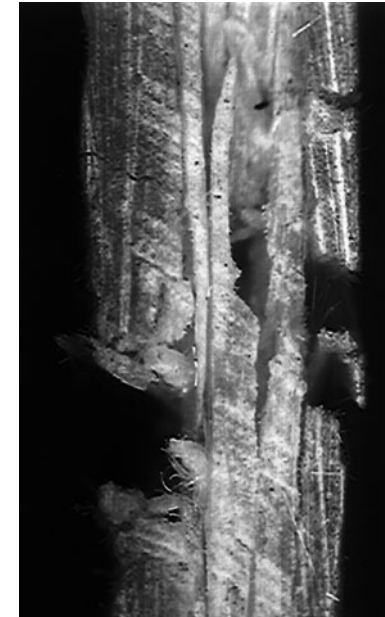
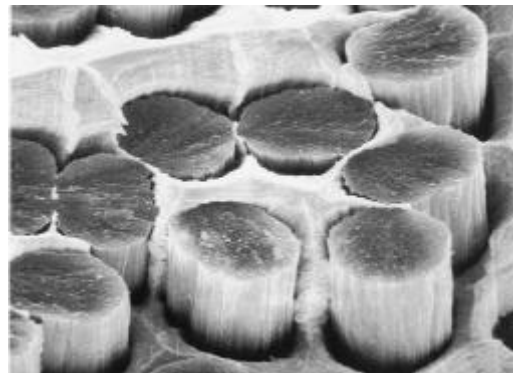
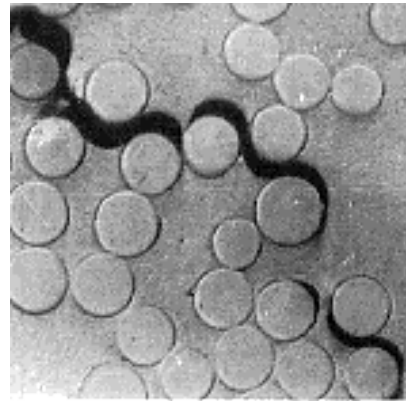


Typical failures of composites (damage mode)

- Matrix failure (brittle matrices)
- Fiber-matrix debonding (soft matrices)
- Delamination

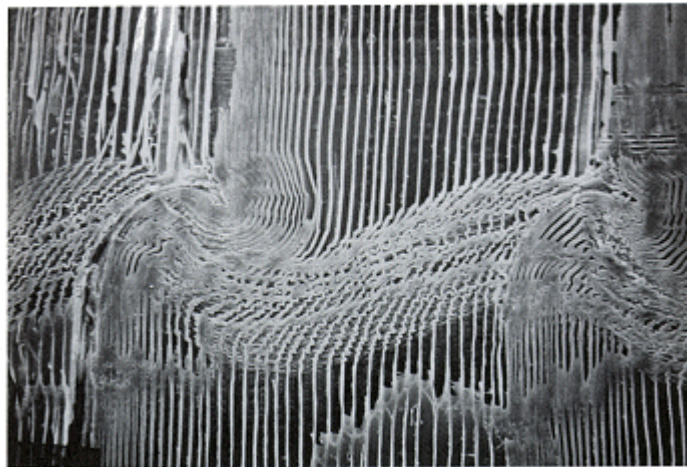
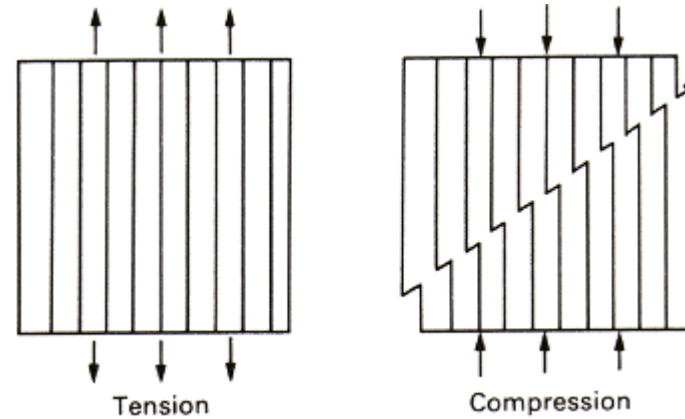


S. Pinho. Modelling failure of laminated composites using physically-based failure models. PhD thesis, Imperial College London, 2005.



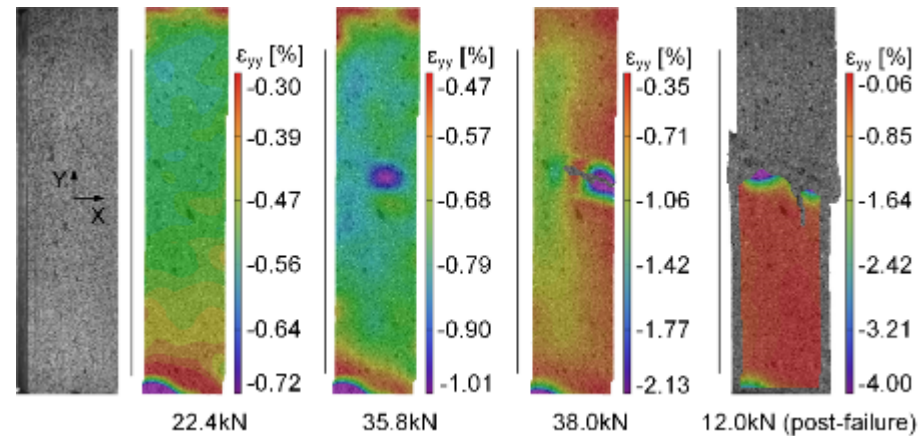
Typical (compressive) failure of composites

- Kinking & crushing



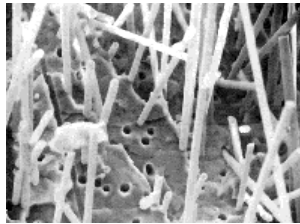
Typical (compressive) failure of composites

- Kinking & crushing

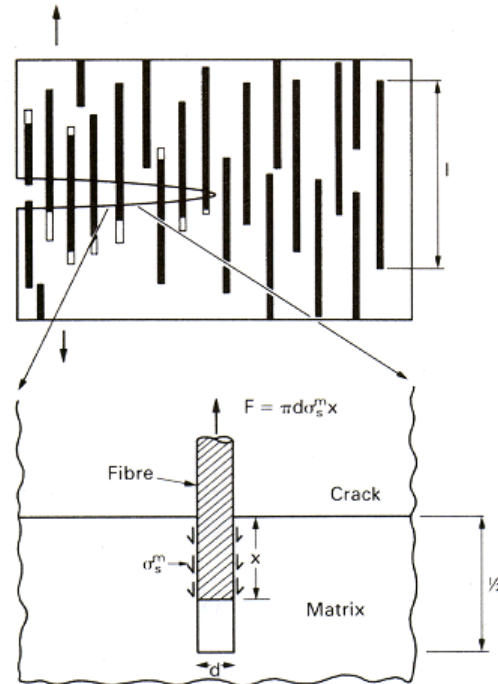
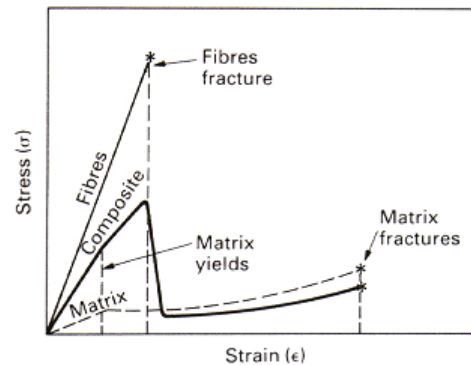


Failure at the micro- and macroscale

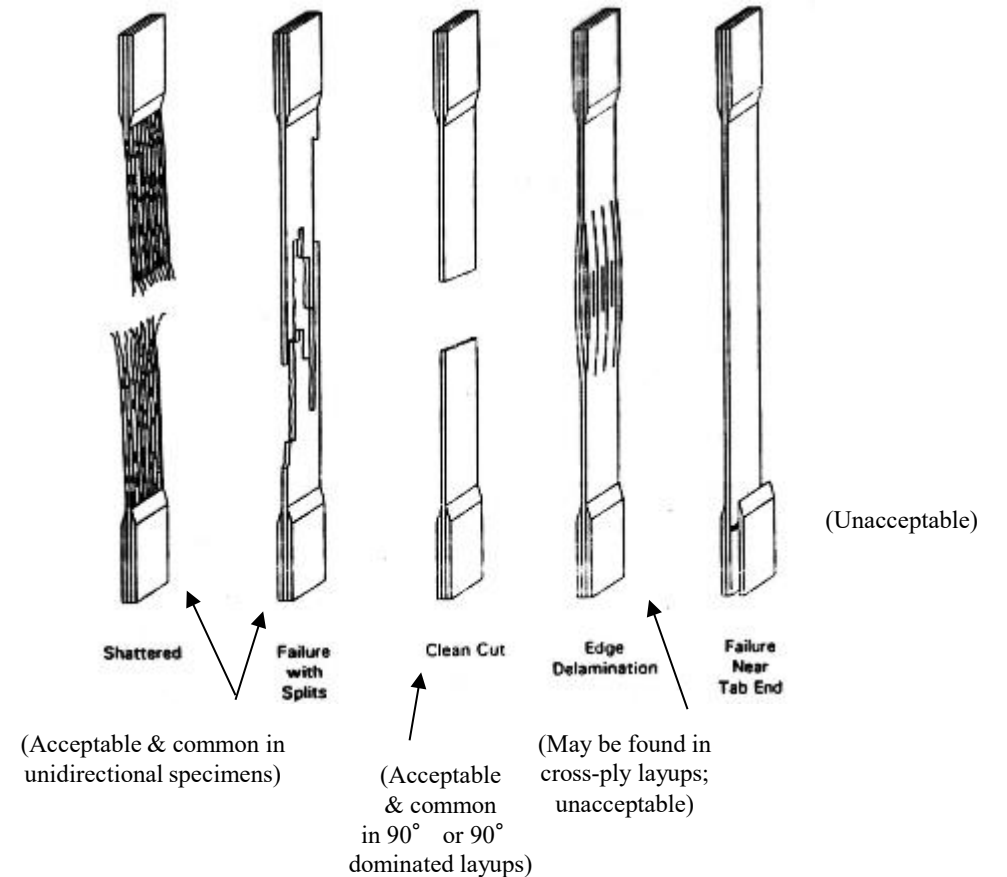
Fiber pull-out & fiber failure



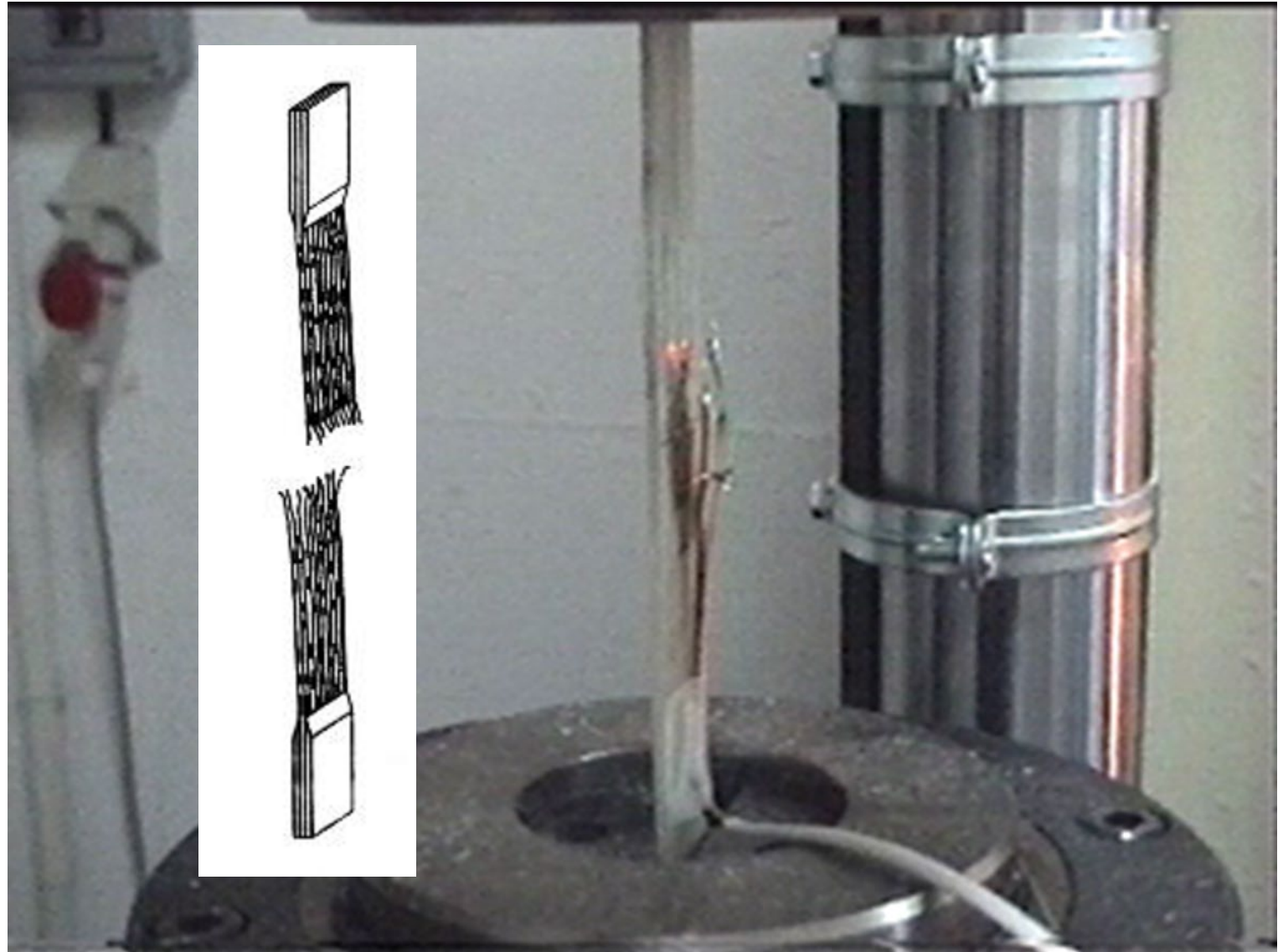
Stress strain curve of a composite



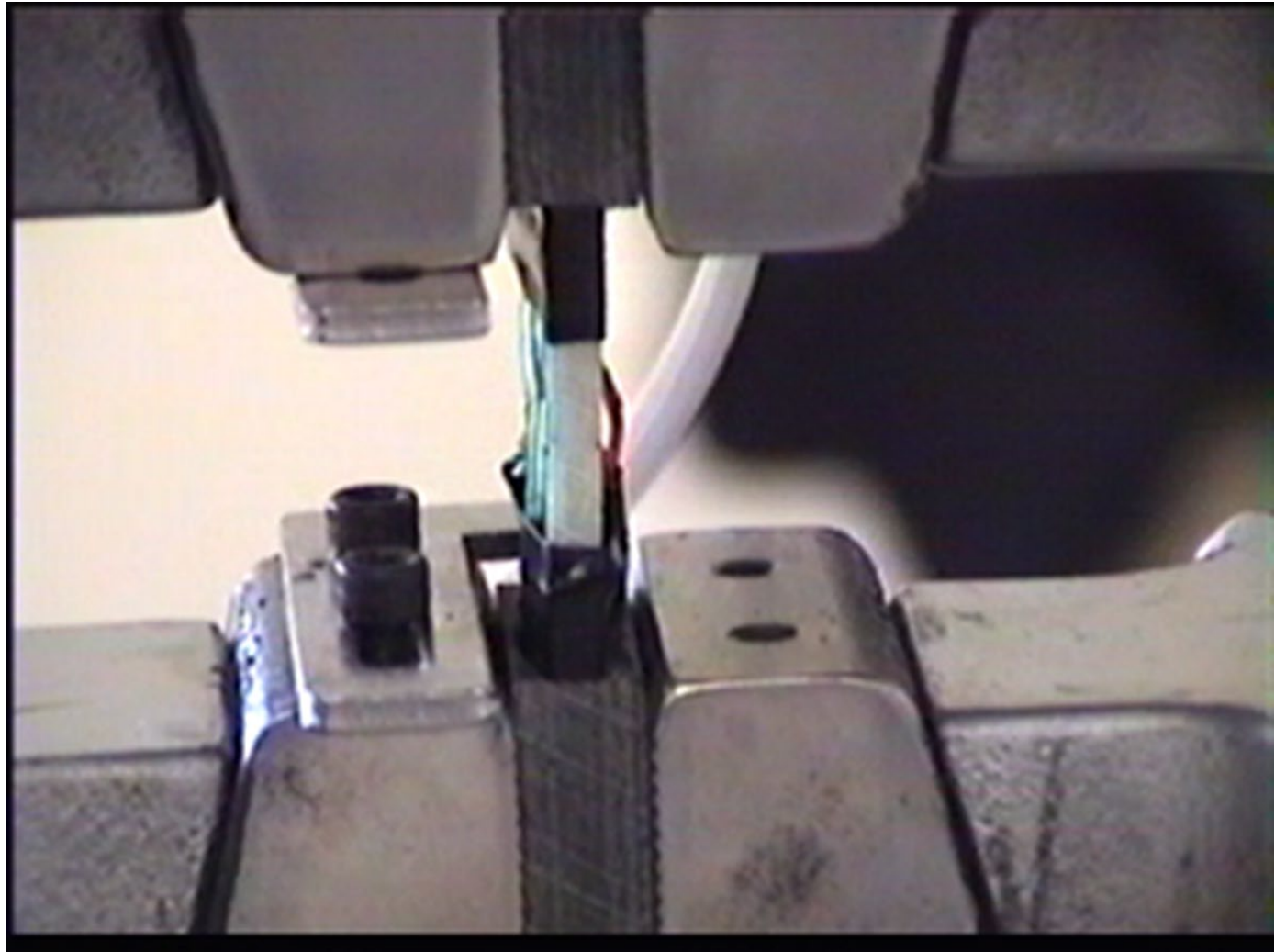
Typical Failure Modes in Straight-Sided Coupons



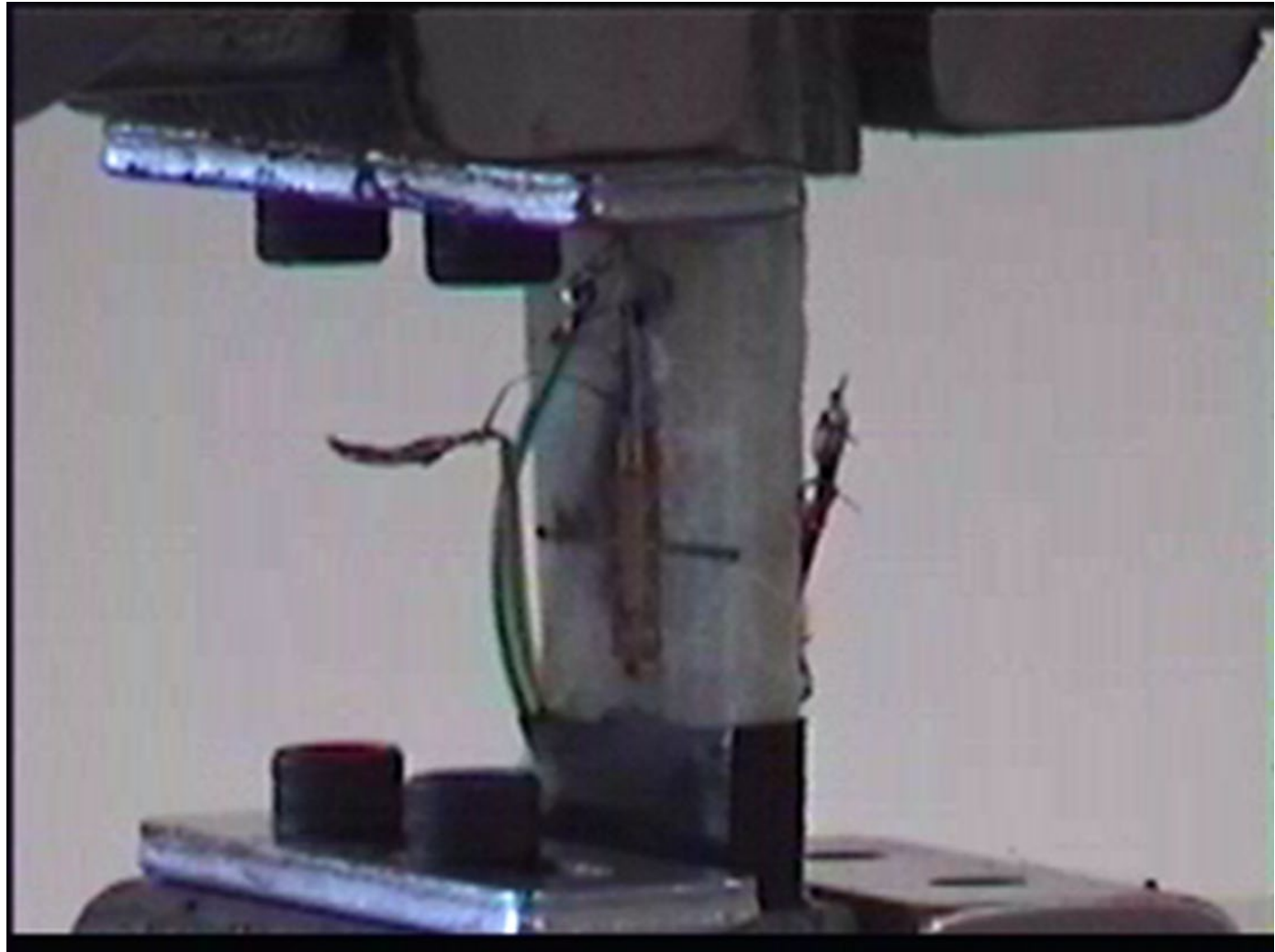
Unidirectional
specimen
tensile failure

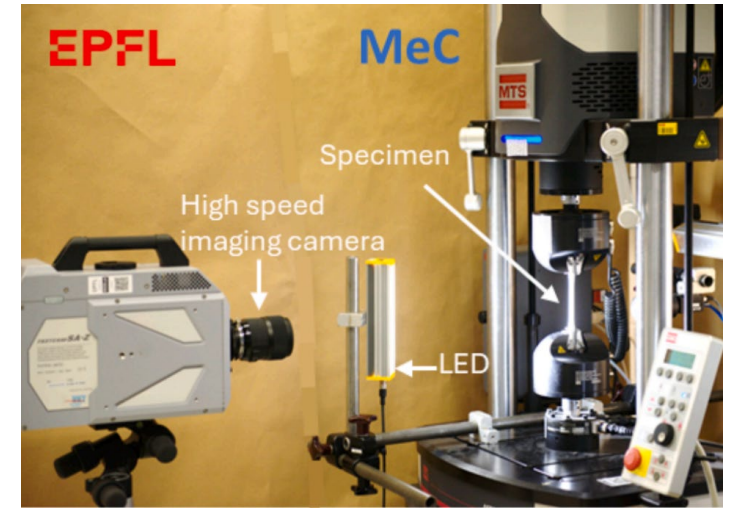
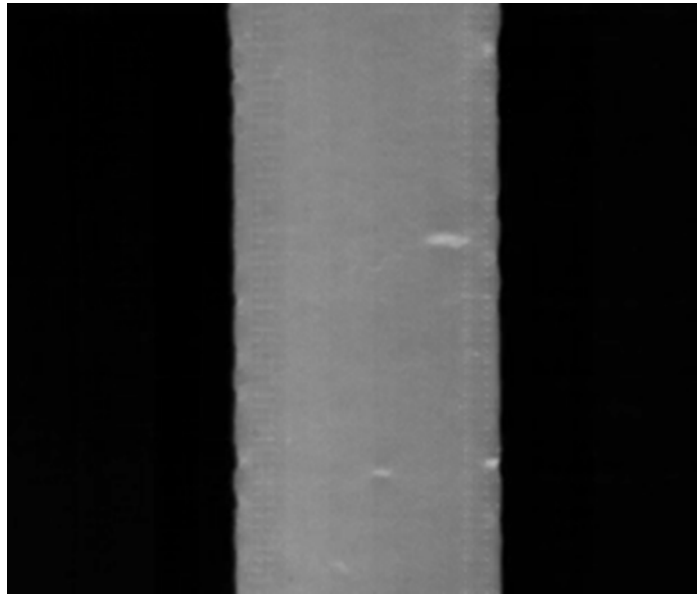
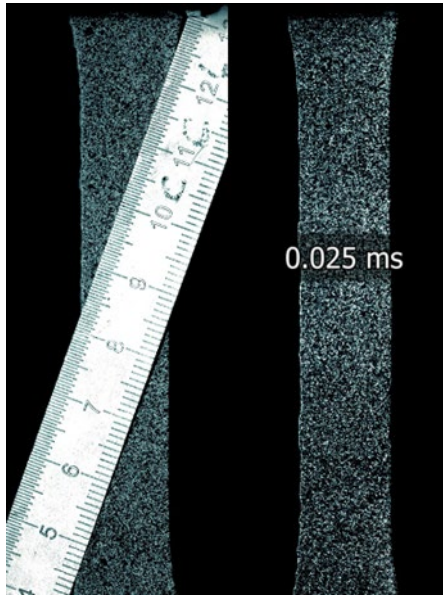


Unidirectional
specimen
compressive
failure



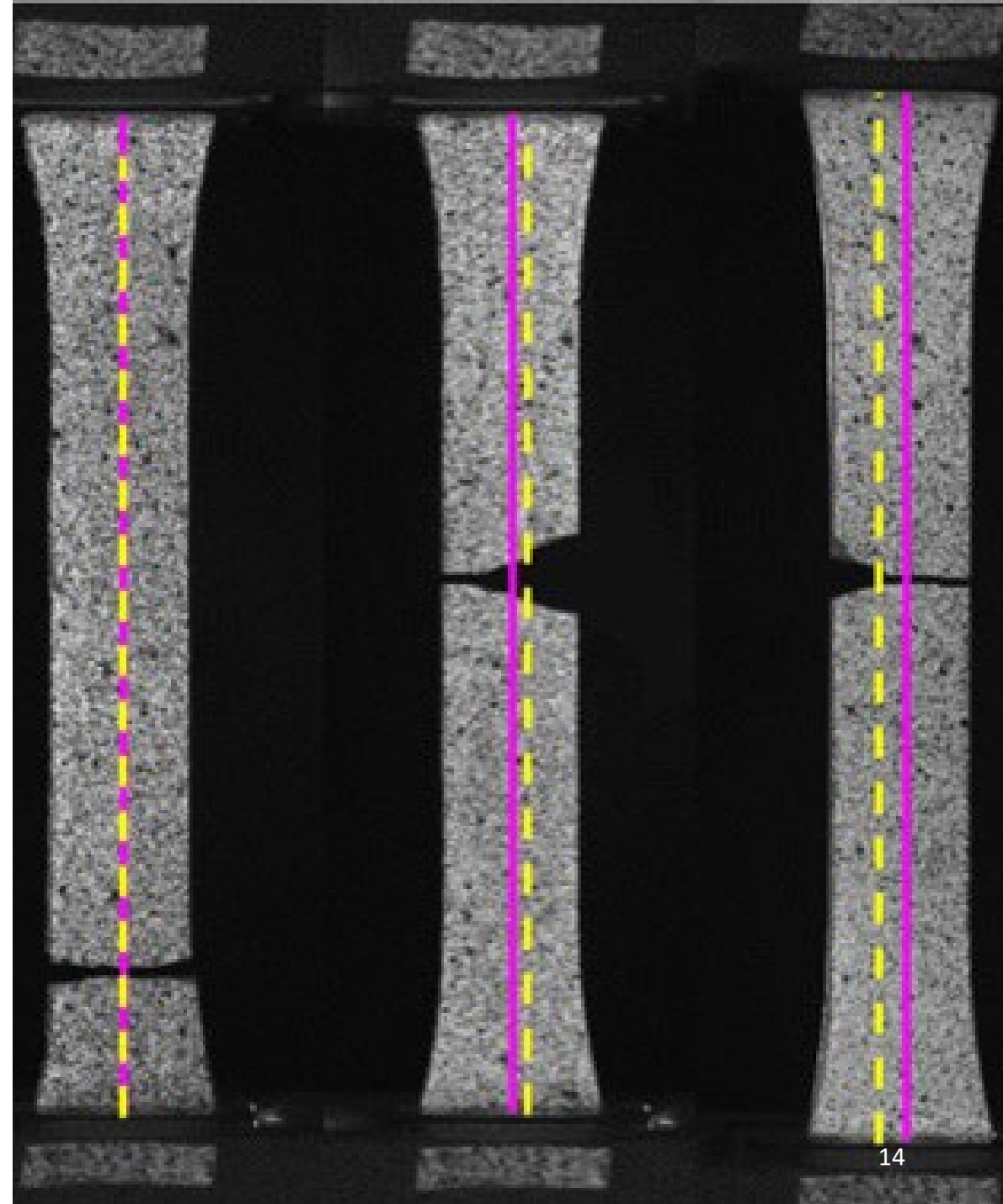
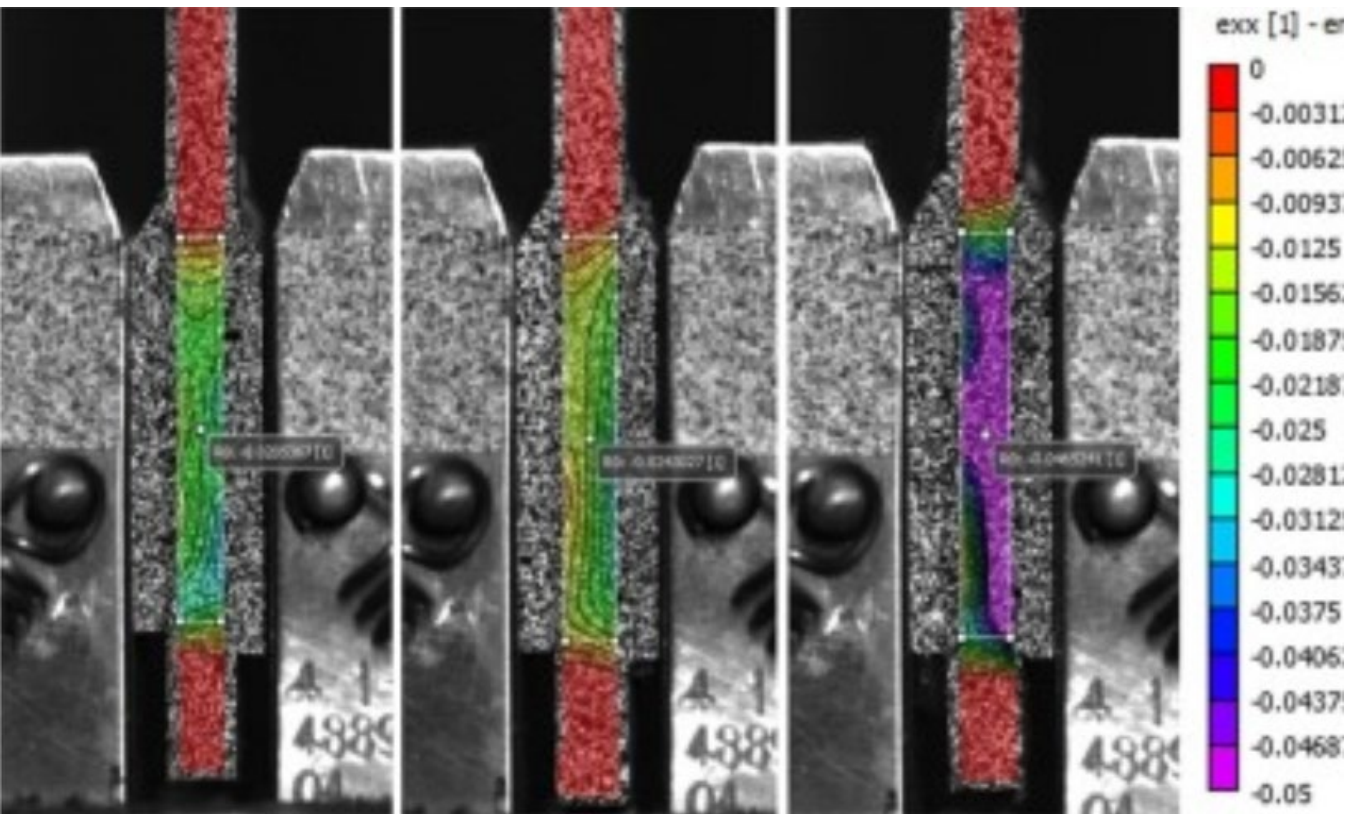
Unidirectional
(short)
specimen
tensile failure





- Viscoelastic materials

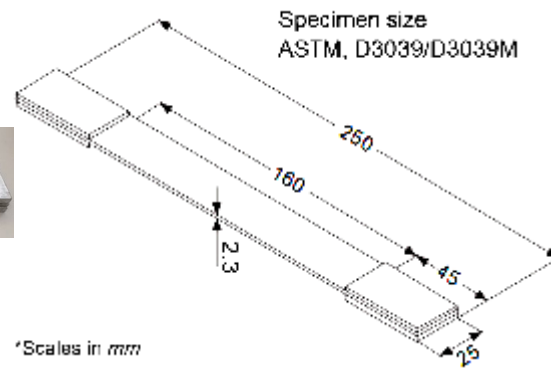
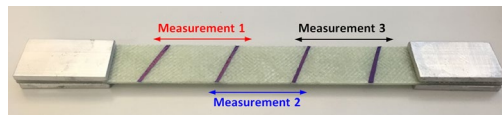
- Sensitivity in preparation and loading



Experimental investigation Specimens and manufacturing

Fabrication:

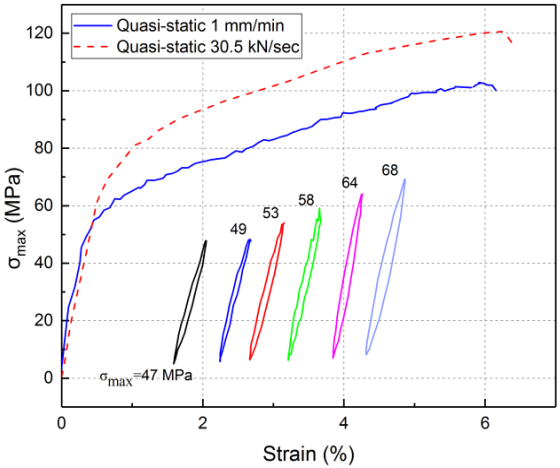
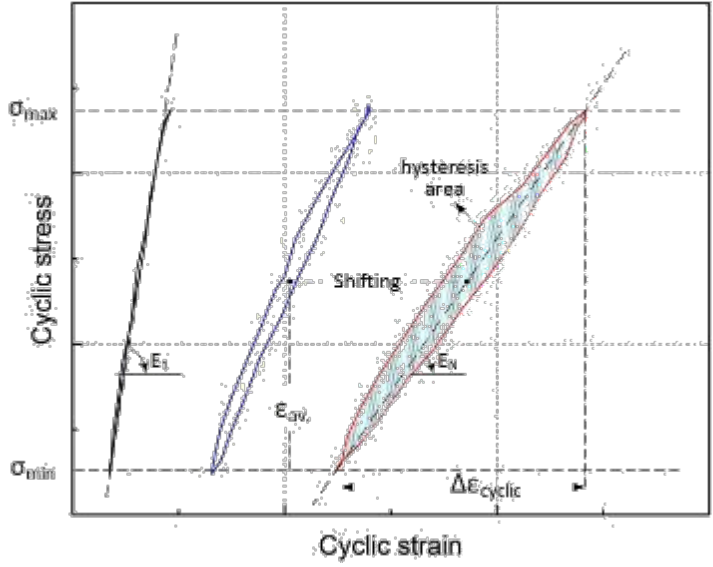
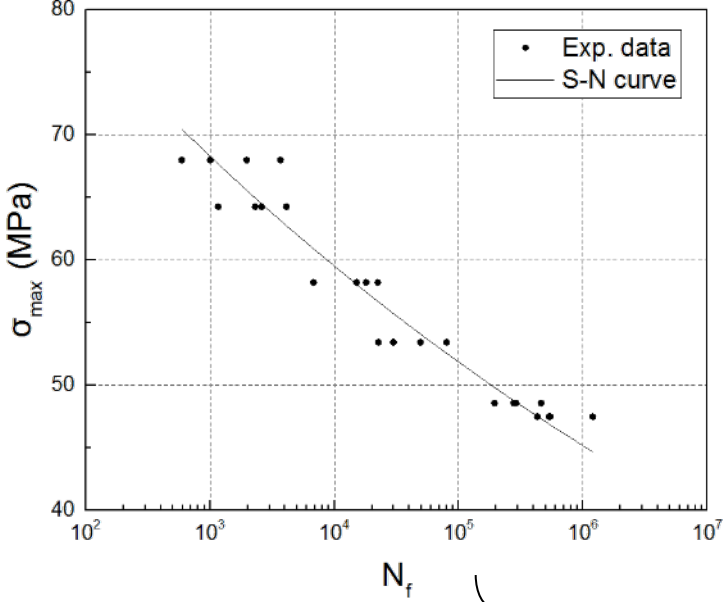
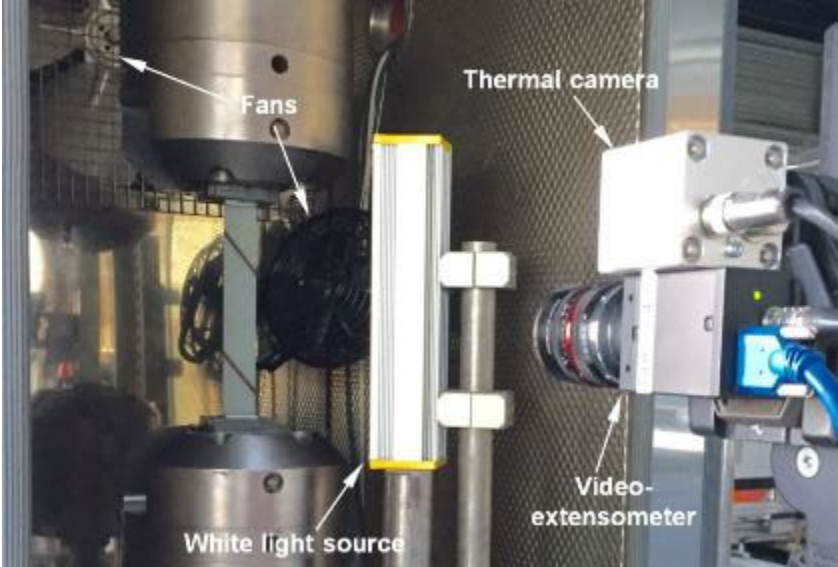
- ❖ Used material: Glass/epoxy (GFRP)
- ❖ Specimen layup (± 45)_{2s}
- ❖ Vacuum assisted hand-lay up technique



Burn off test:

- ❖ 62% Fiber content
- ❖ 38% Matrix content

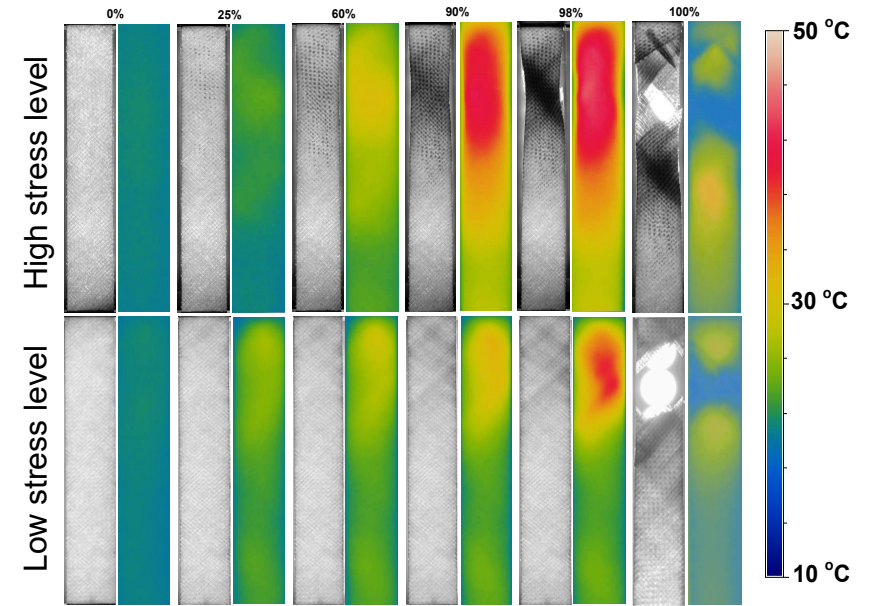
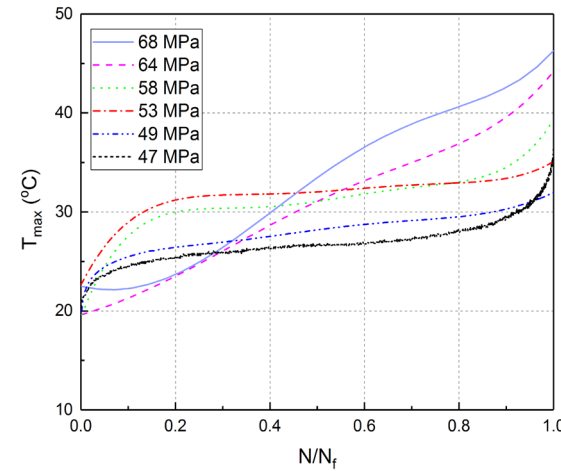
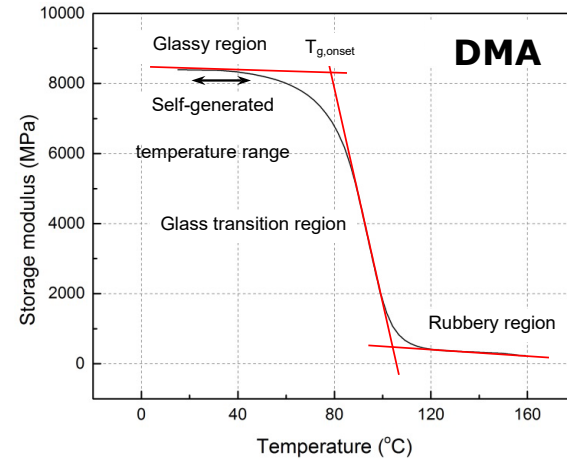
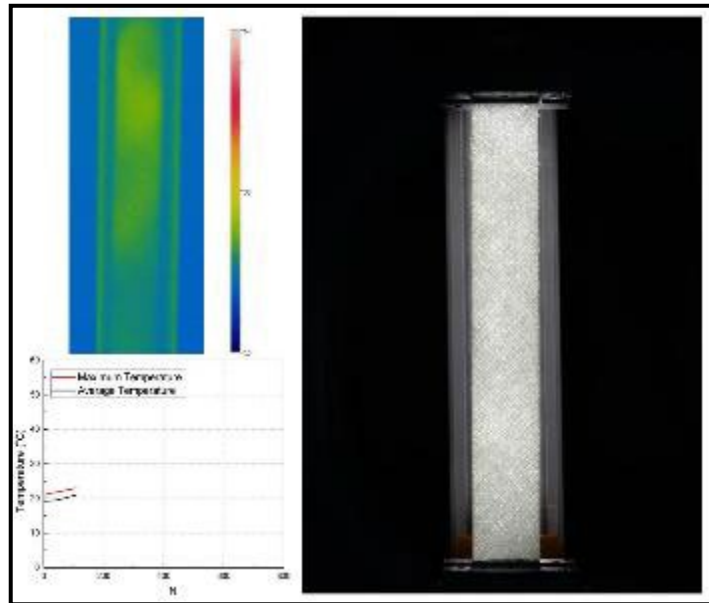
Experimental results: QS and fatigue



Fatigue behavior

- Fatigue life
- Fatigue stiffness
- Hysteresis loop area
- Cyclic creep: $(\epsilon_{max} + \epsilon_{min})/2$

Thermal behavior/observation ($R=0.1$)

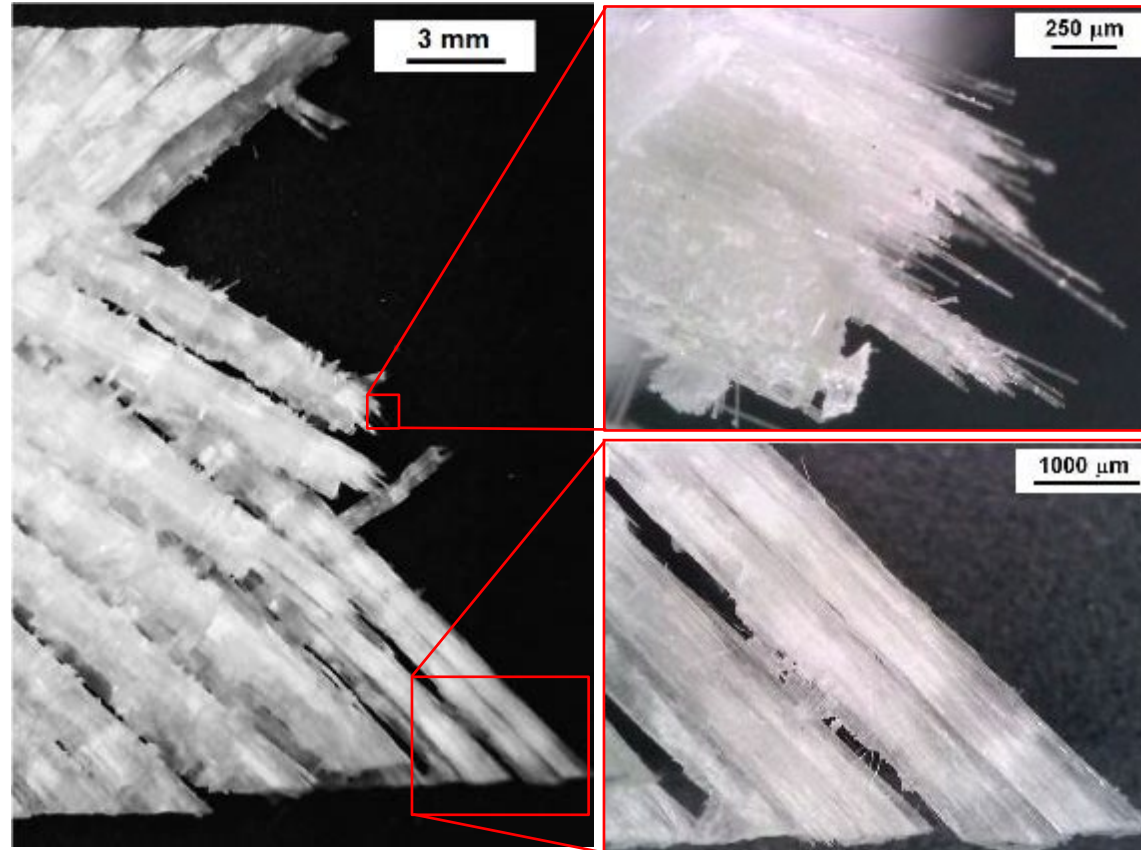


Damage development with stress level

Failure mechanisms



Digital handheld microscope

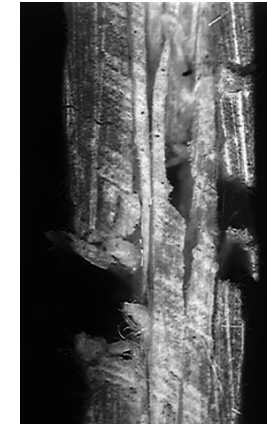
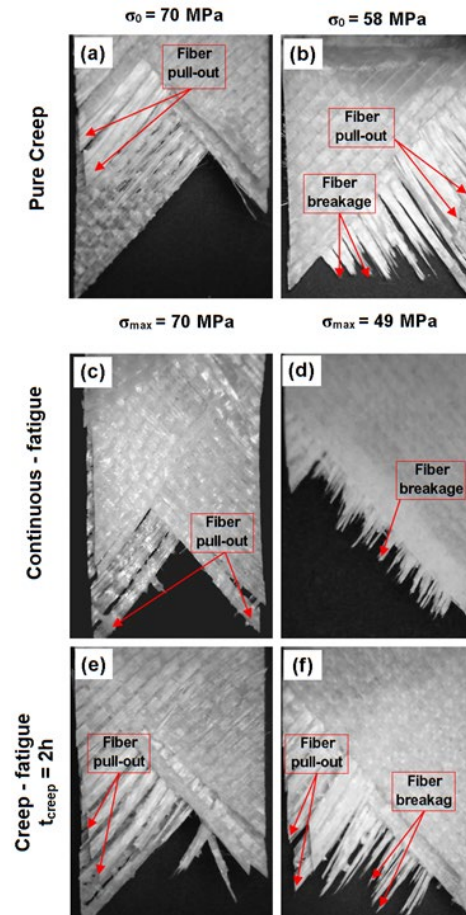
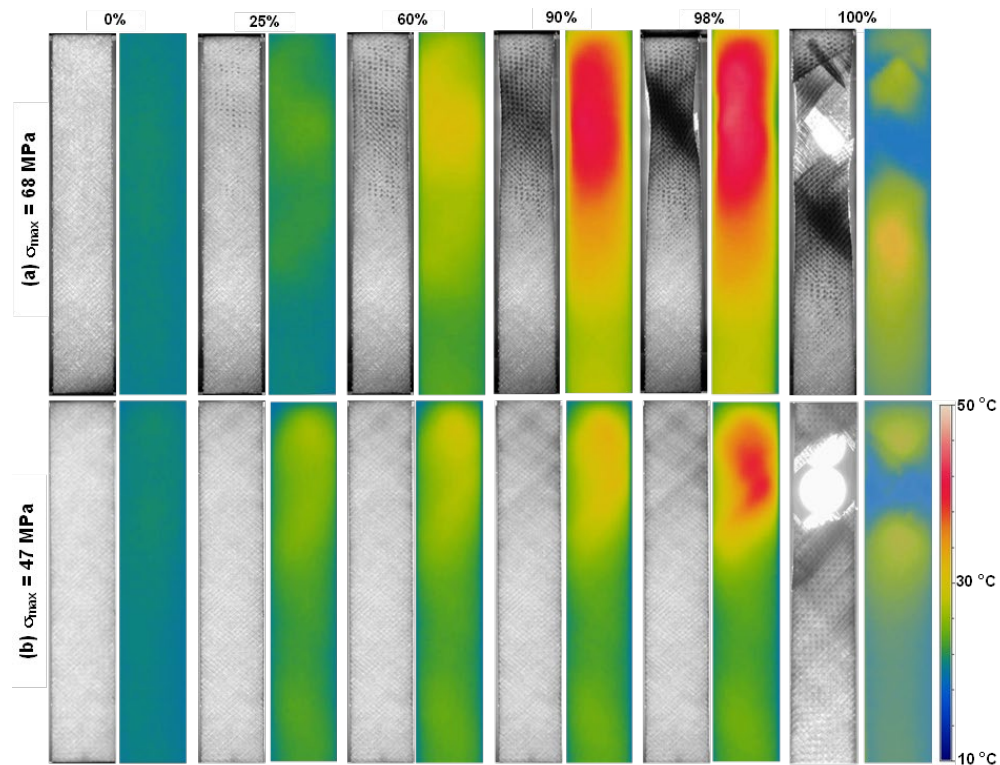


1. Fiber breakage

2. Fiber pull-out

Damage monitoring

Optical microscopy



Lateral view of
 90° off-axis specimen
under $R = -1$ at 38 MPa

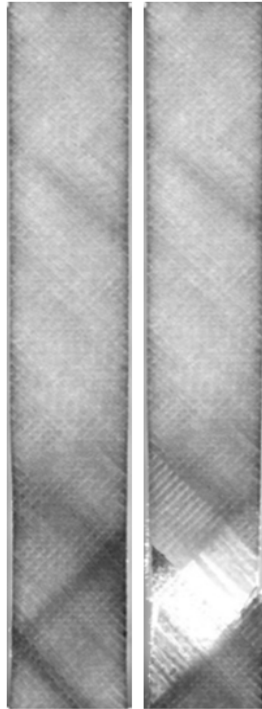
Damage monitoring (smaller scale)

Optical inspection

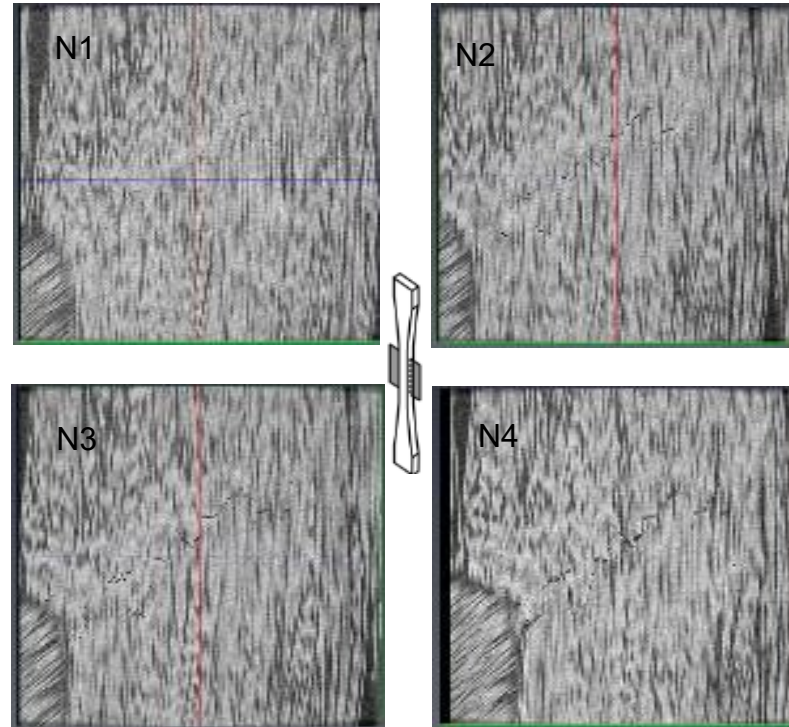
(a) Continuous-fatigue

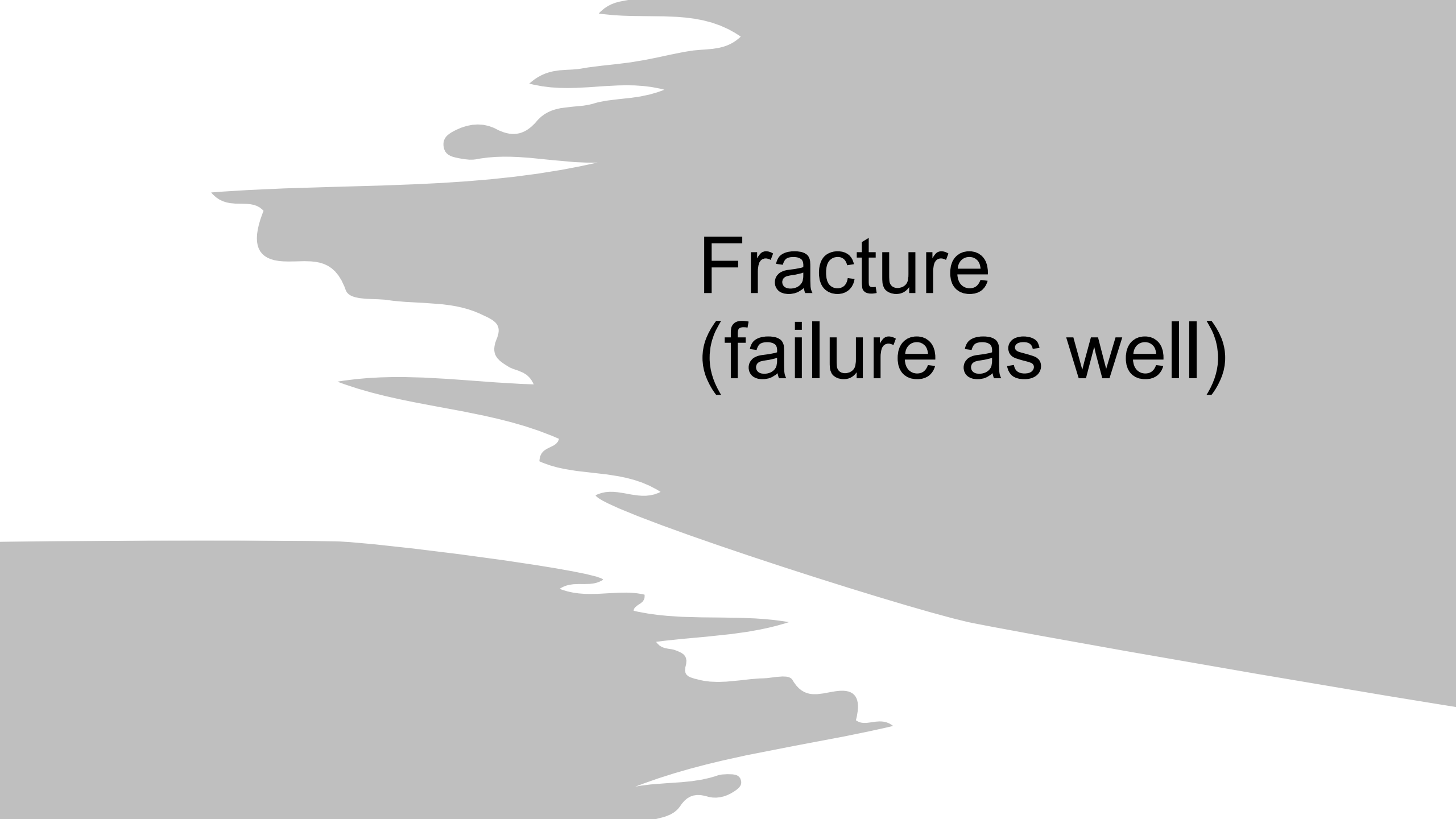


(b) Pure creep



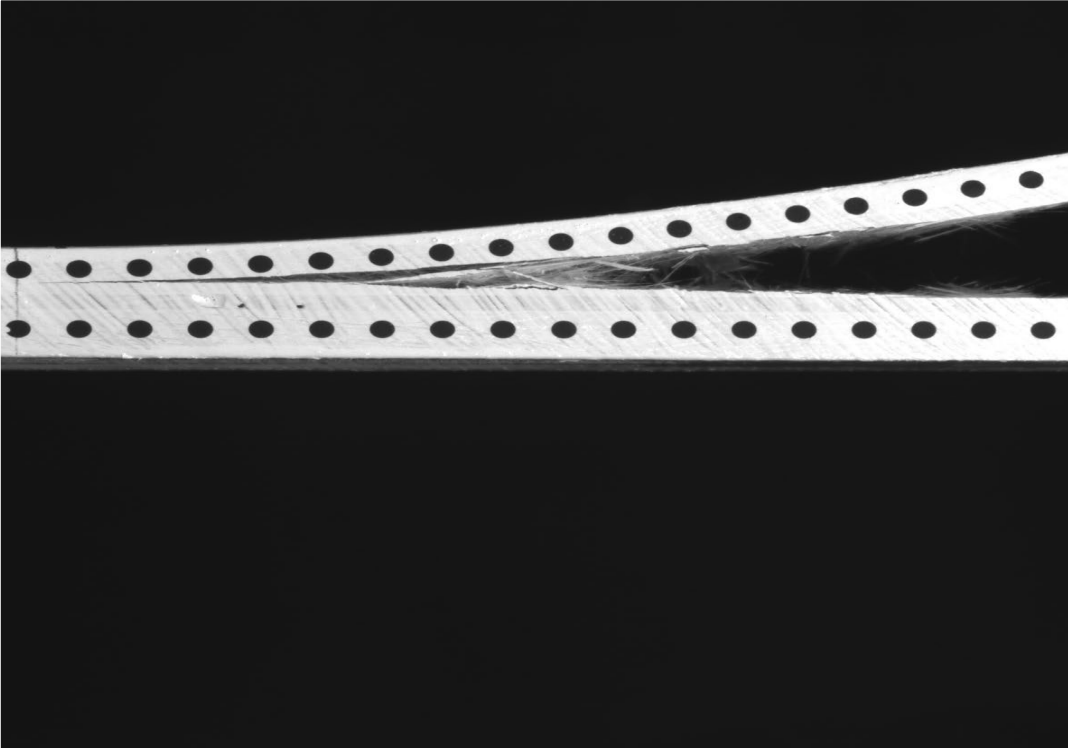
Magnetic tomography



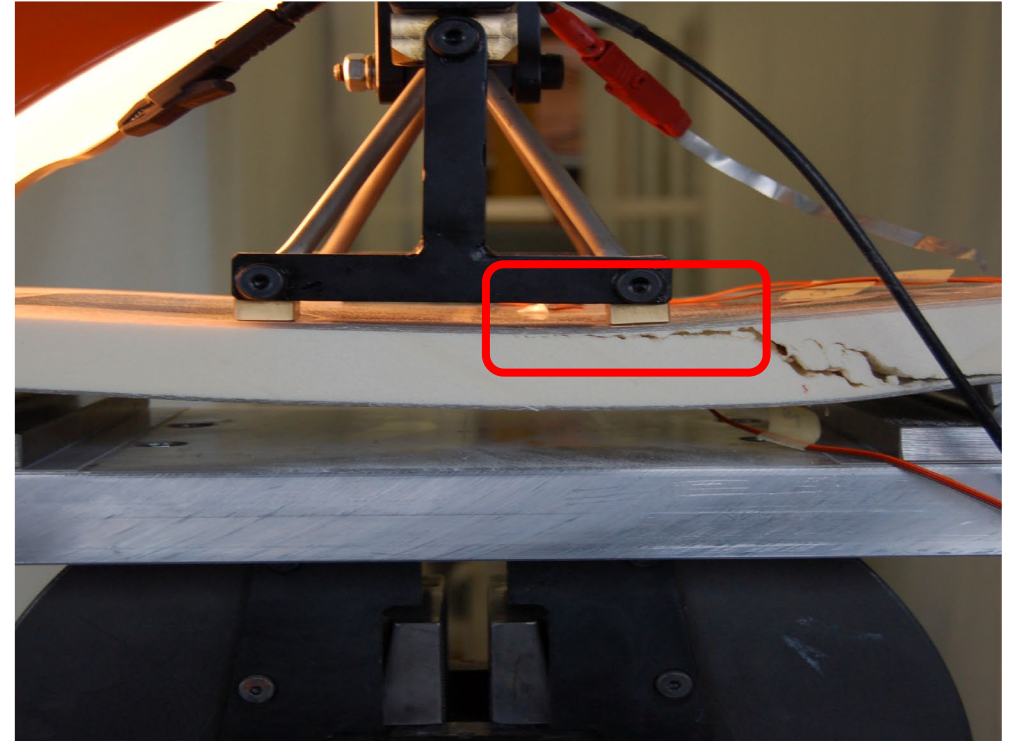


Fracture
(failure as well)

Fracture

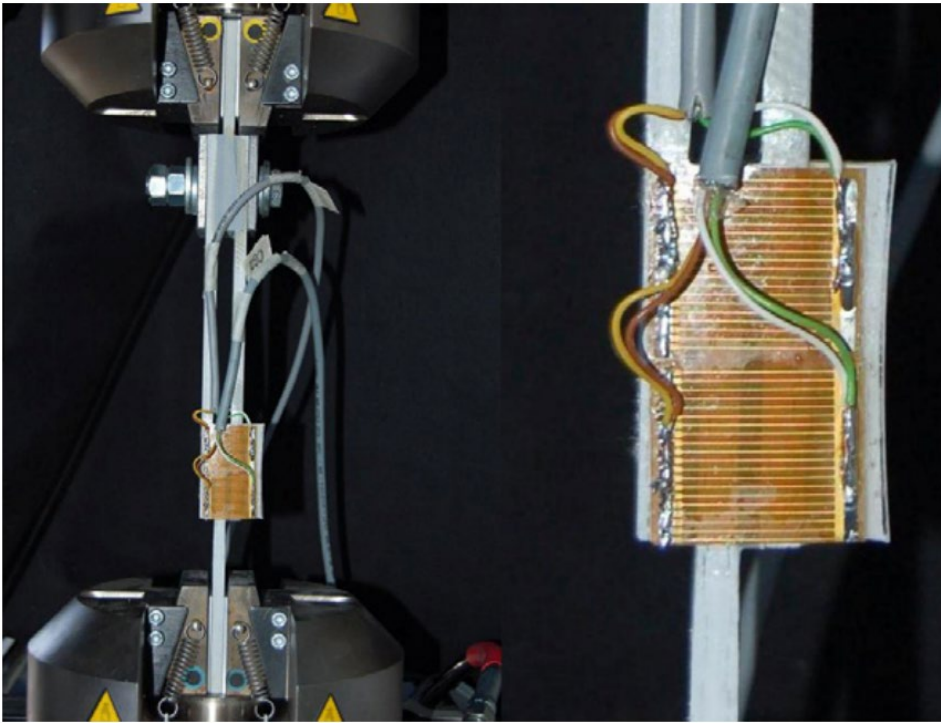


Delamination

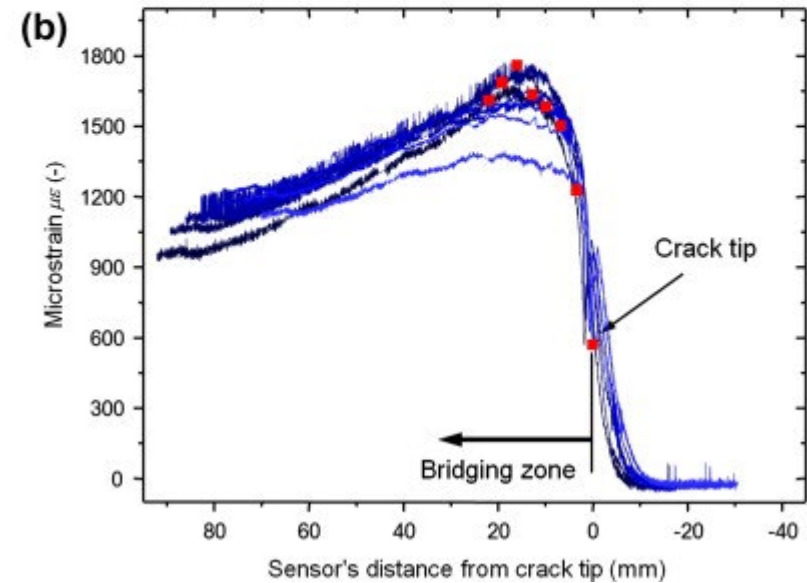
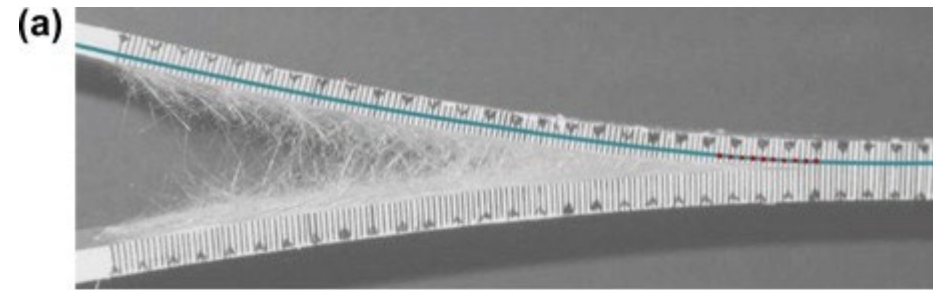
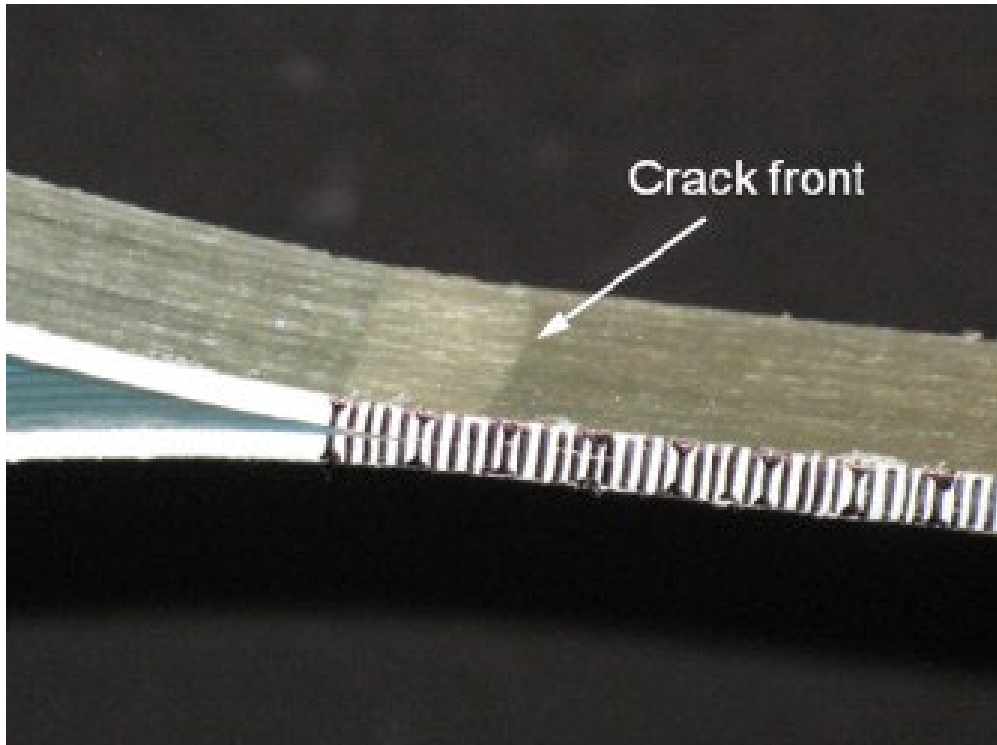


Debonding

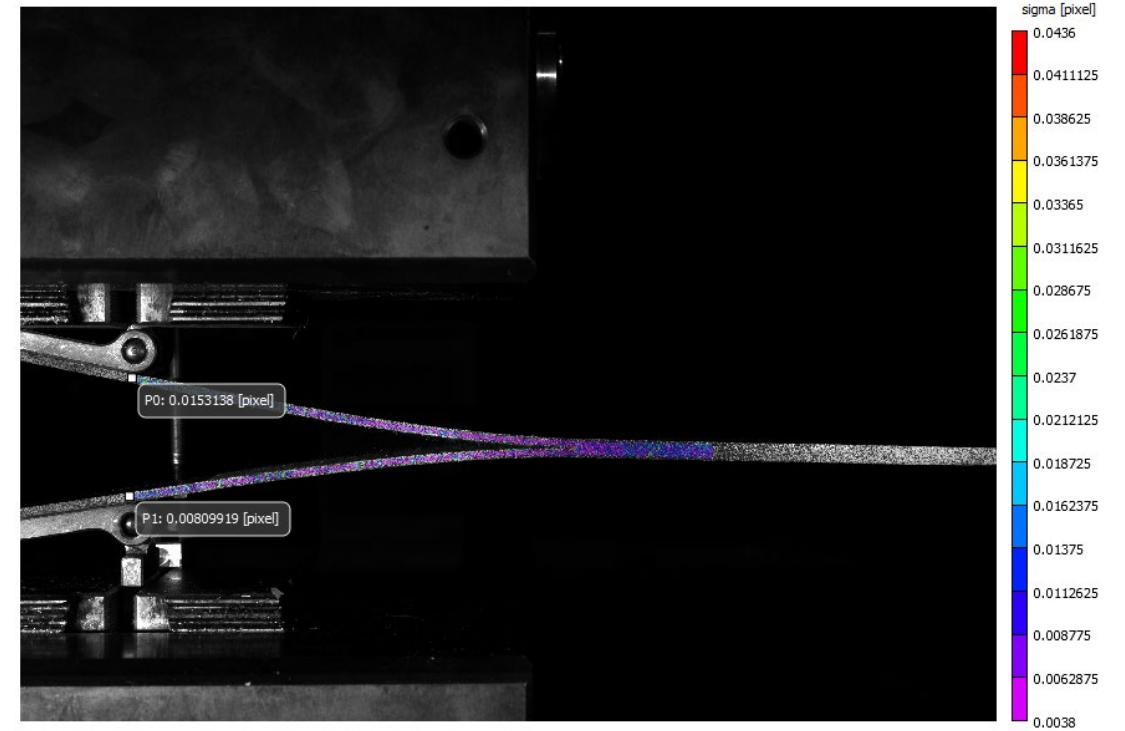
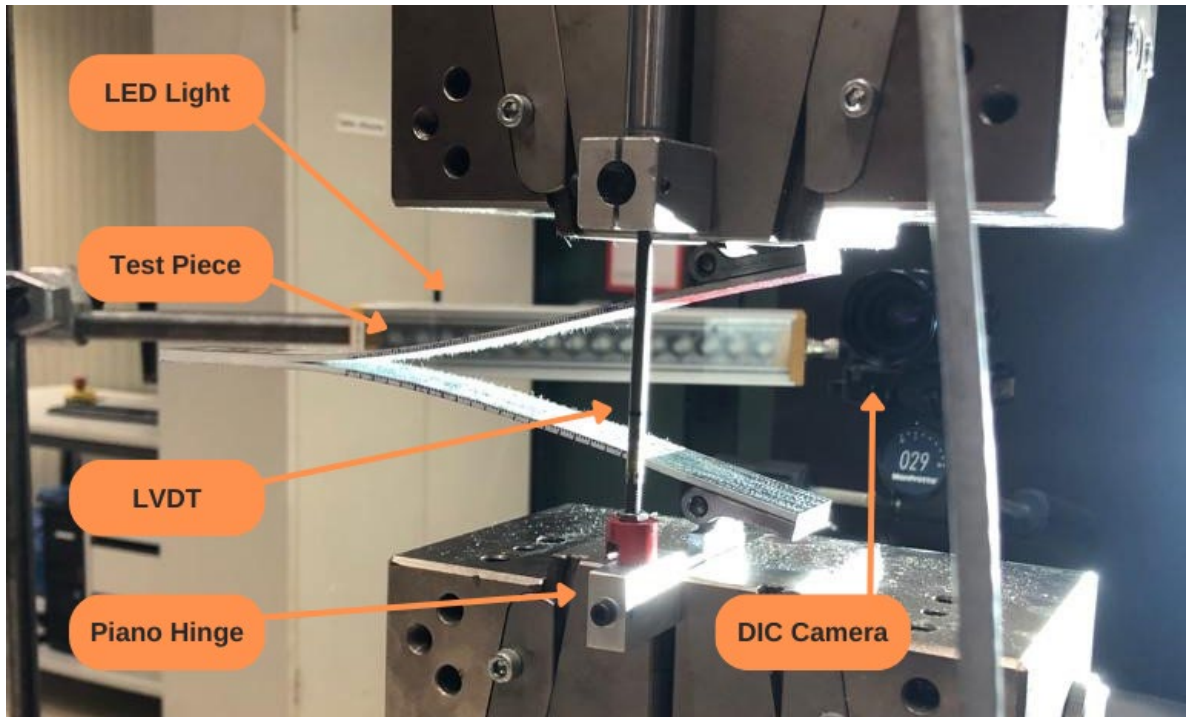
Damage monitoring

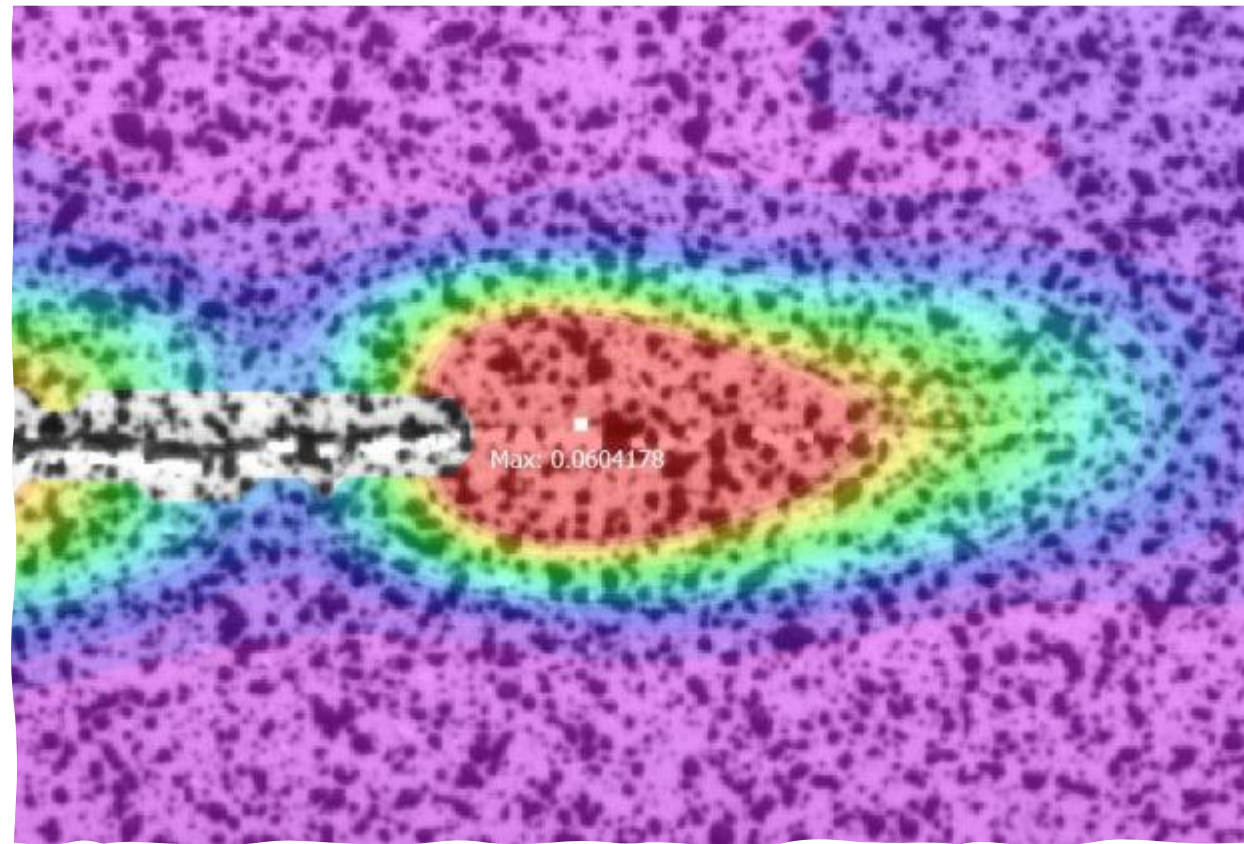
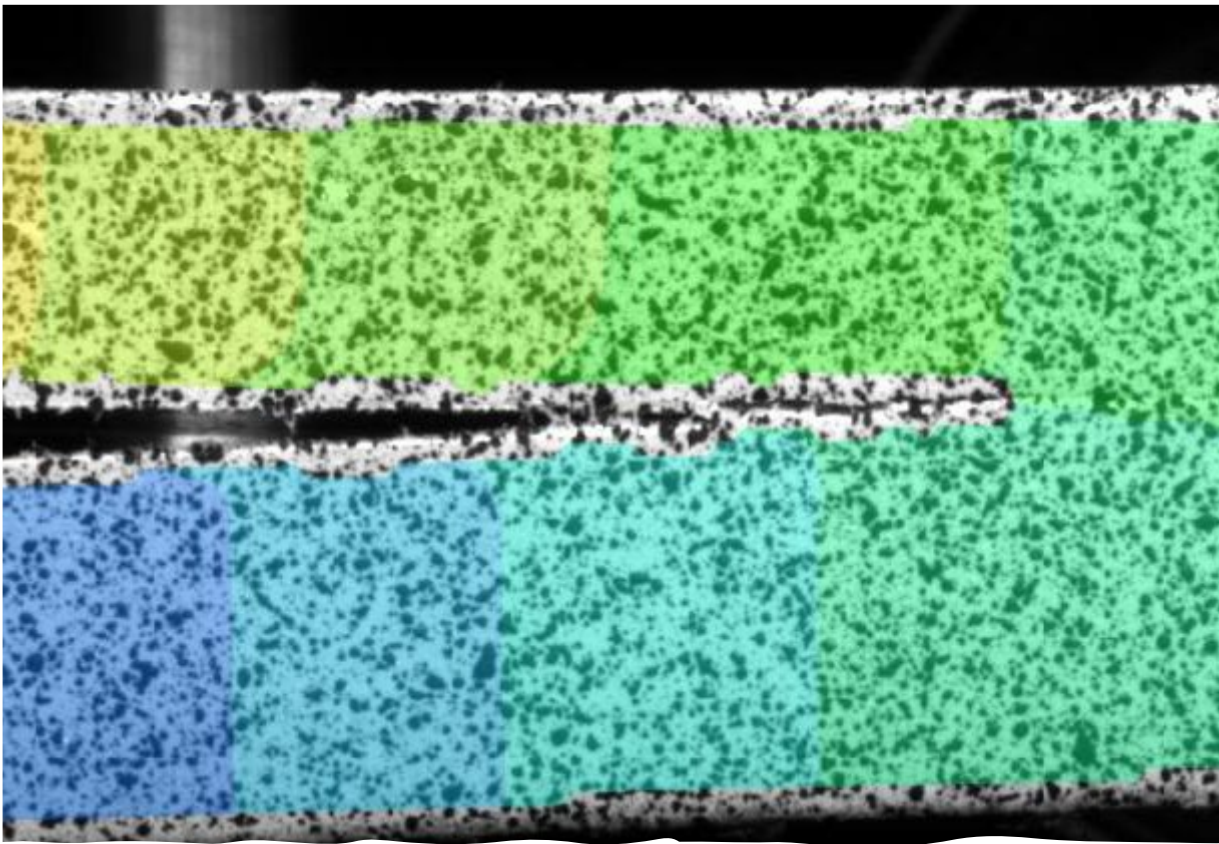


Damage monitoring



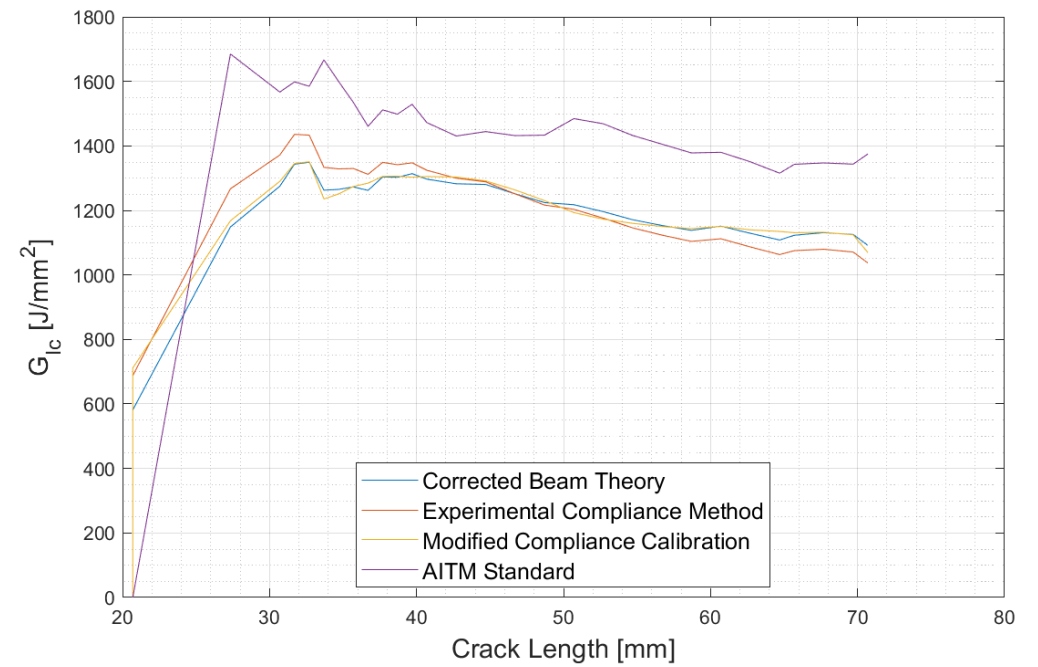
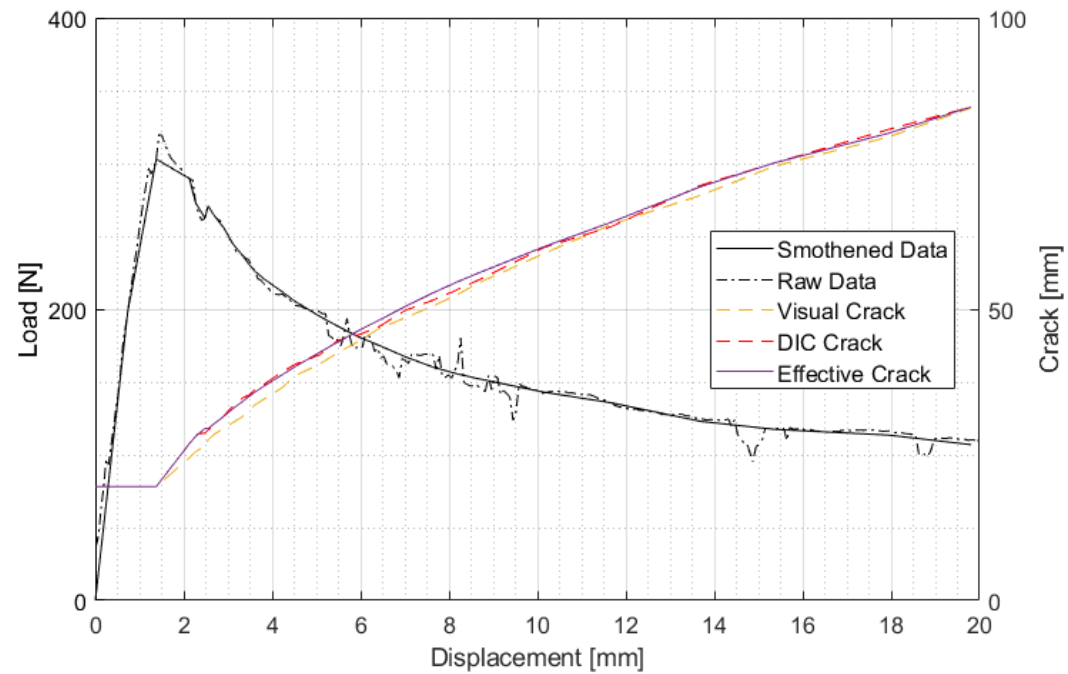
Damage monitoring



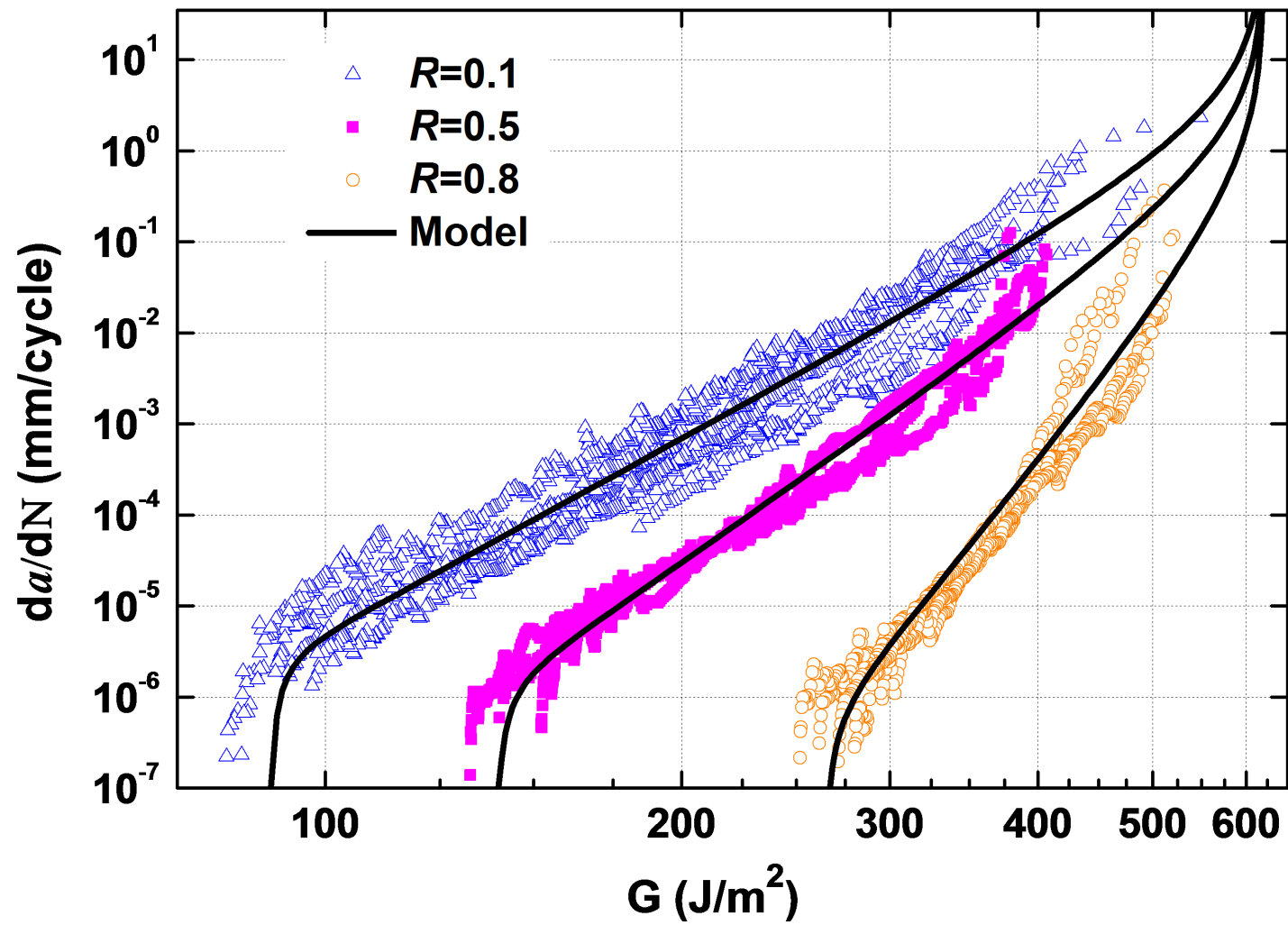


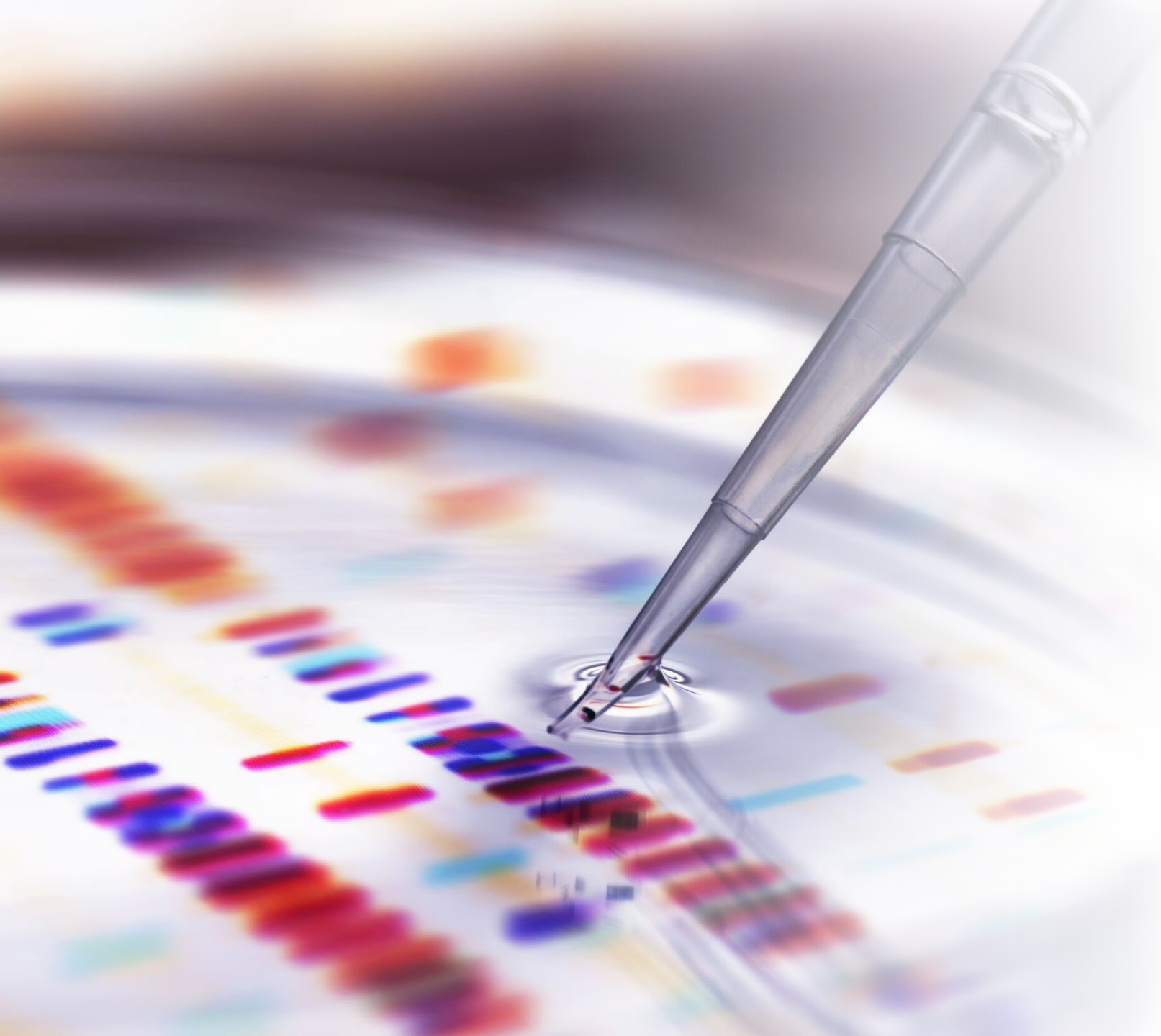
Damage monitoring

Objective (quasi-static)



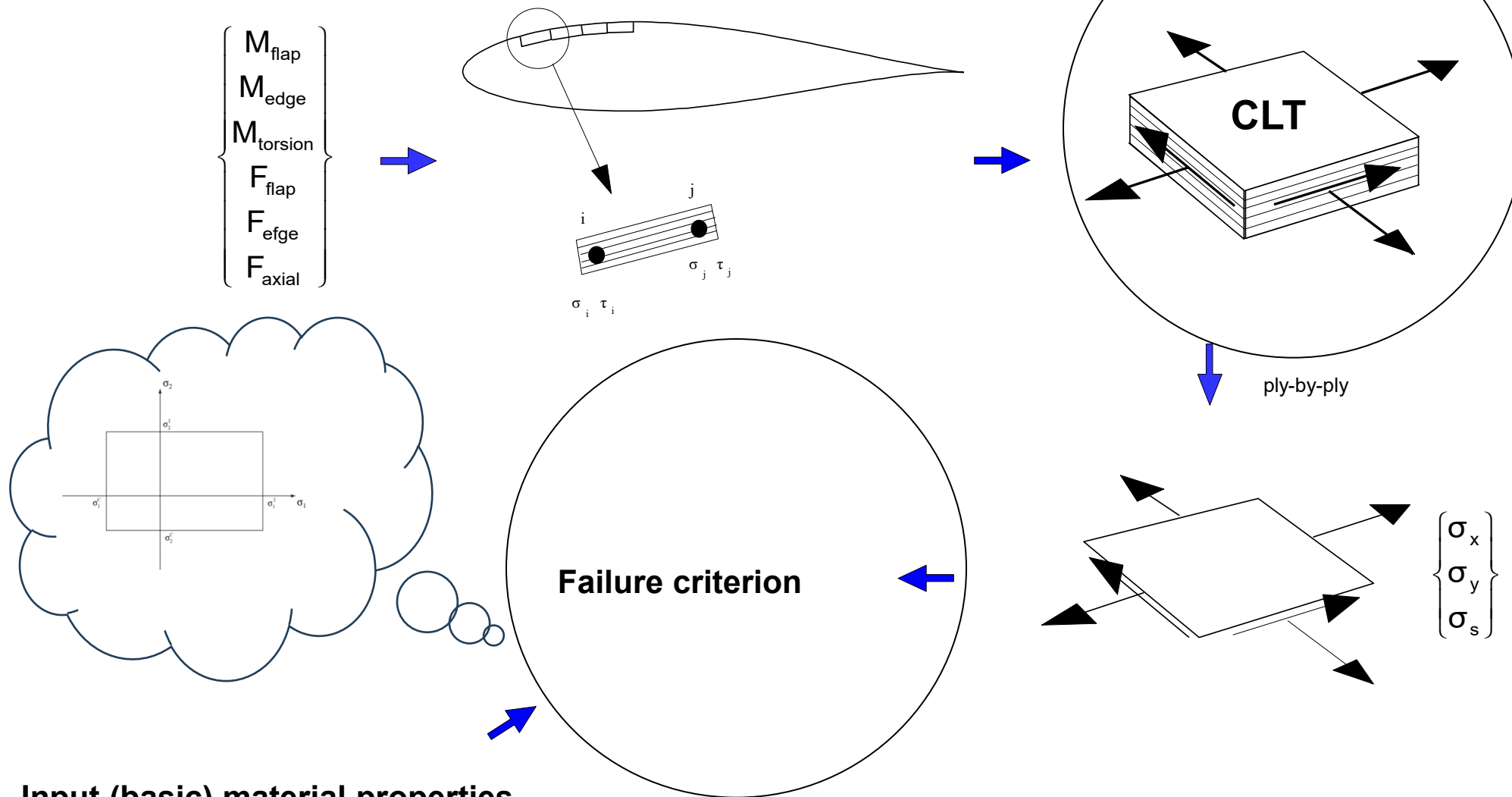
Objective
(fatigue)





Failure analysis

Failure analysis (stress/strain)



Input (basic) material properties
For each layer or...

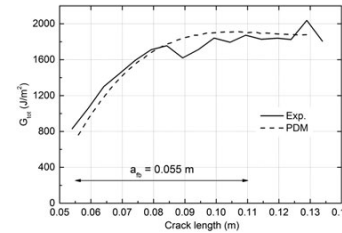
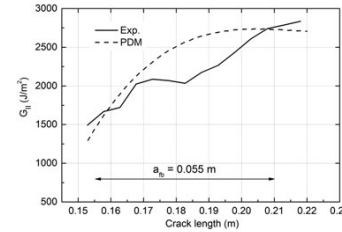
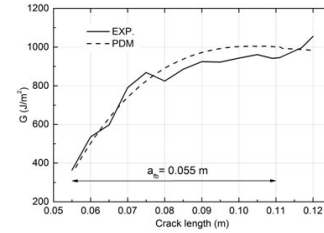
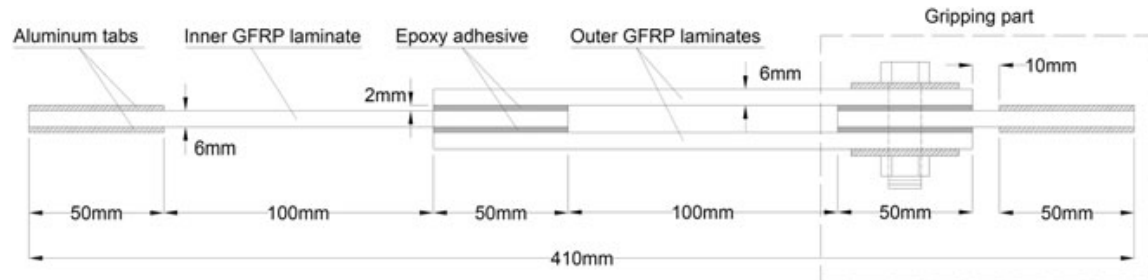
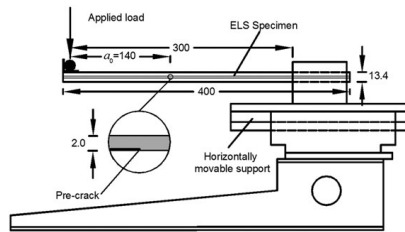
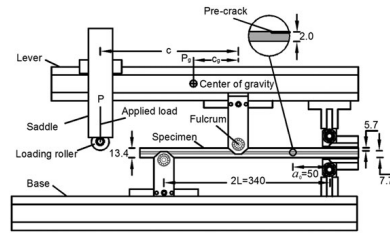
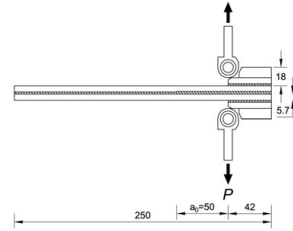
Failure analysis (fracture)

Input:

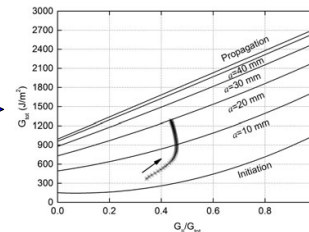
Material



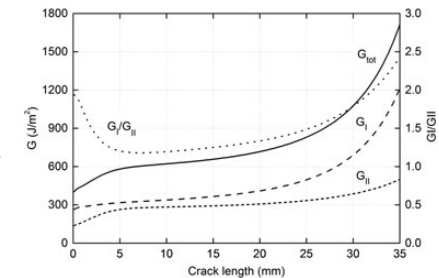
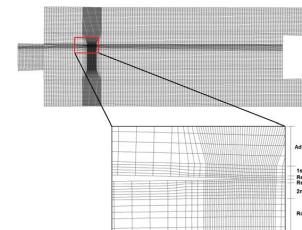
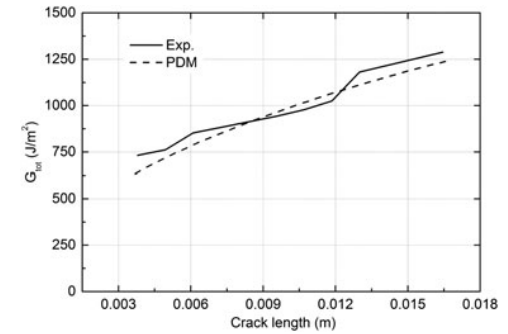
Geometry

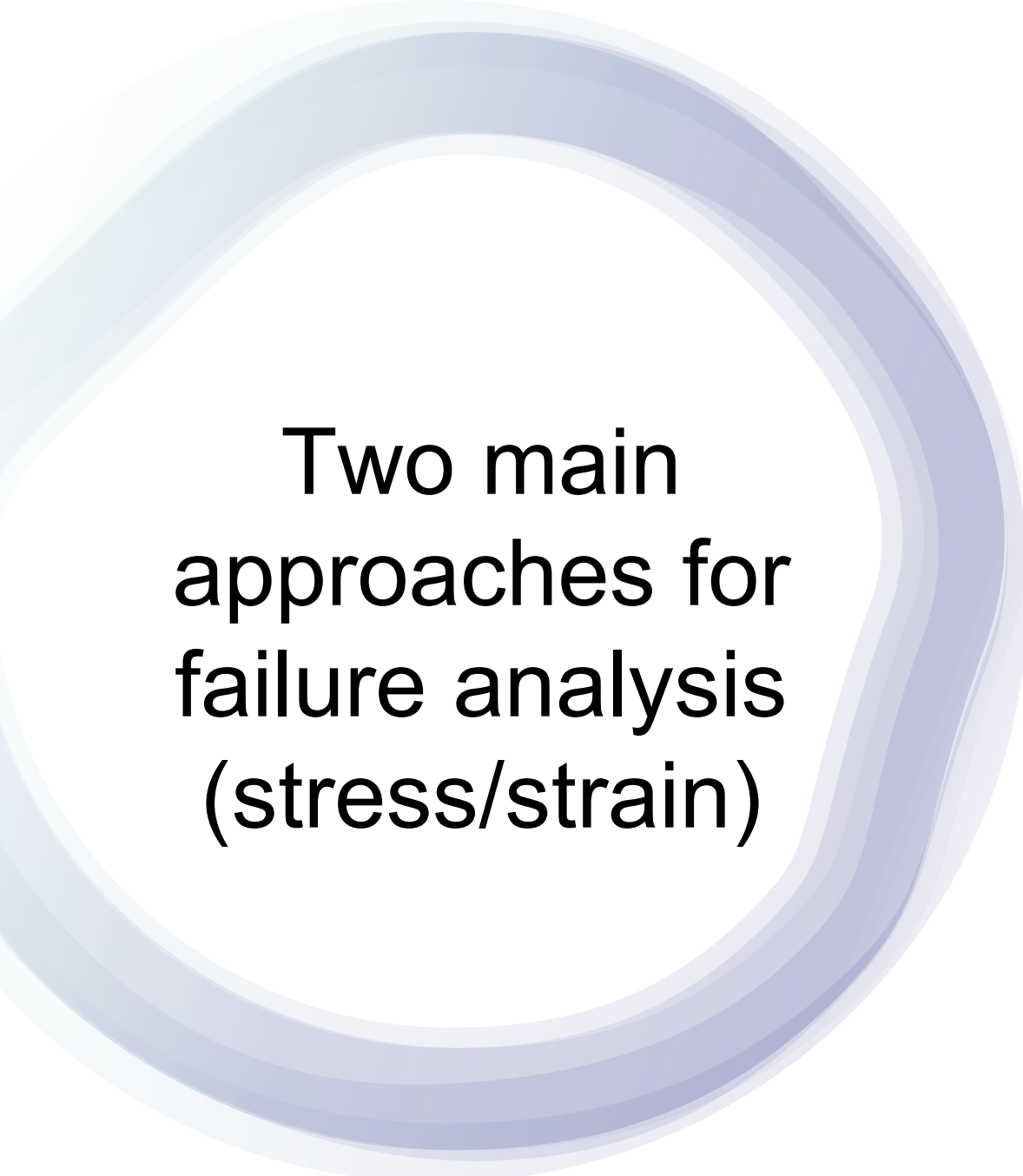


Failure criterion



$$G_{tot} = A \left(\frac{G_{II}}{G_{tot}} \right)^2 + B \left(\frac{G_{II}}{G_{tot}} \right) + C$$





Two main approaches for failure analysis (stress/strain)

- **a) Maximum stress and strain criteria**
 - Not interactive failure criteria associated with failure modes
- **b) Quadratic interaction criteria**
 - Interactive failure criteria that can be associated or not with failure modes

Strength of (isotropic) materials



Estimate the stress at
the time and location
when failure occurs

For conventional
(isotropic) materials:

Maximum tensile,
compressive or shear
stress and observation
about the failure is
enough

Usually, one strength
parameter is enough...

Young's modulus is enough
since **Poisson's ratio**
equals to **0.3** and the
uniaxial tensile strength
suffice since **shear strength**
is taken to be about **50-60%**
of the tensile.

Maximum stress/strain

Related to the maximum normal stress theory by Rankine and the maximum shear stress theory by Tresca.

The stresses acting on a point are resolved into the **principal normal stresses** or the **maximum shear stresses**. Failure is predicted if any of the stress components is equal to or exceeds the corresponding strength of the material.

$$-X' < \sigma_1 < X$$

$$-Y' < \sigma_2 < Y$$

$$-S < \sigma_6 < S$$

OR for the maximum
strain theory:

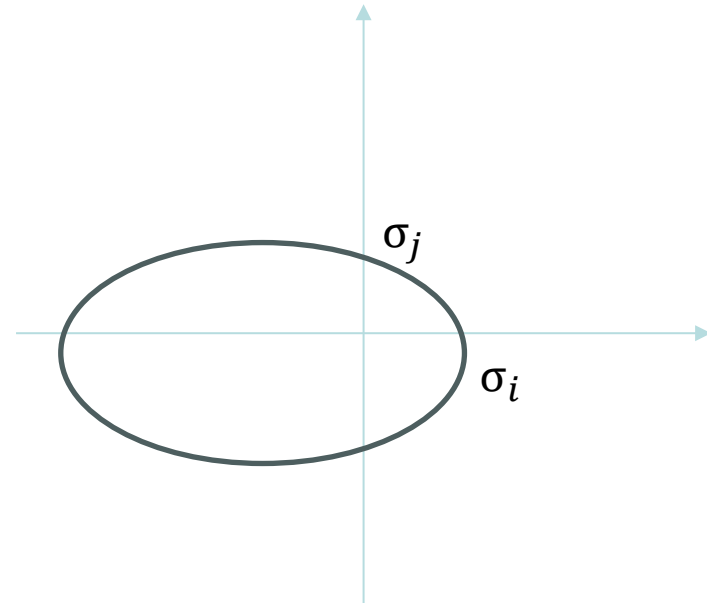
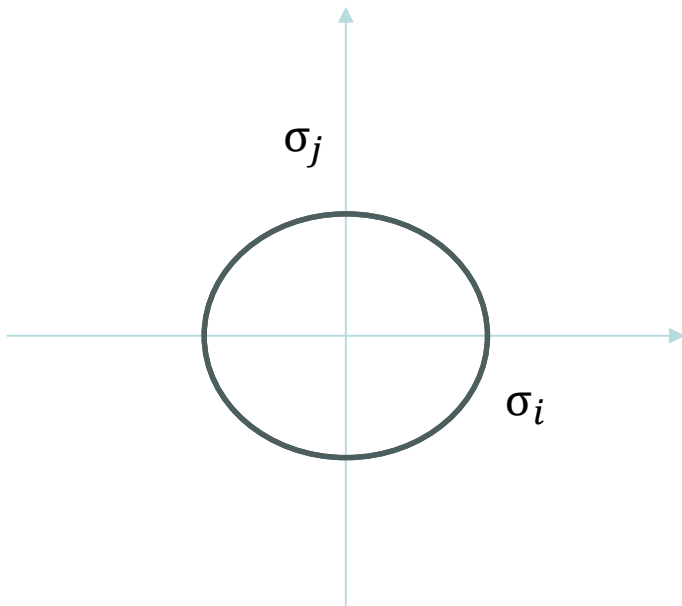
$$-\left(\varepsilon_1^c\right)_{ult} < \varepsilon_1 < \varepsilon_{1ult}^t$$

$$-\left(\varepsilon_2^c\right)_{ult} < \varepsilon_2 < \varepsilon_{2ult}^t$$

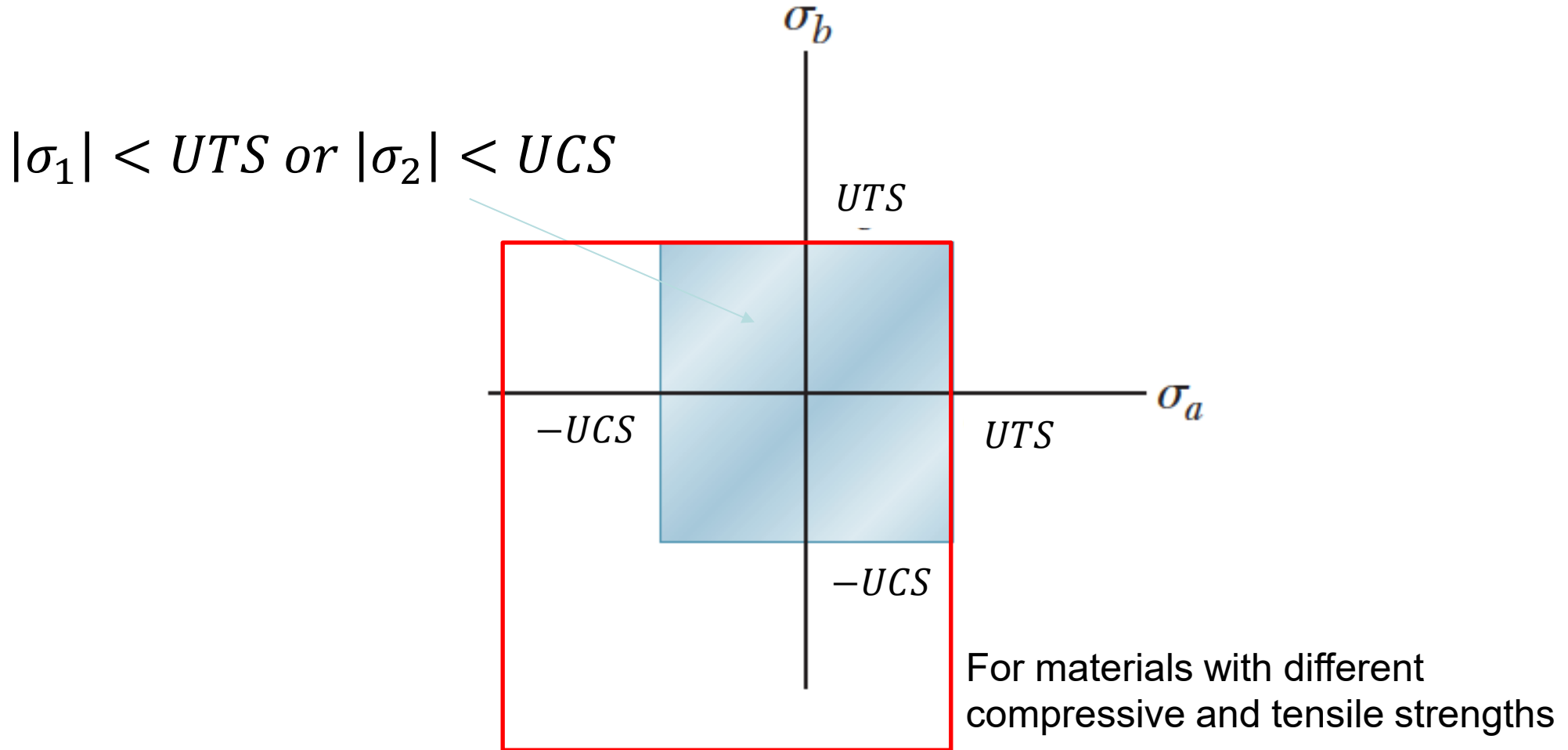
$$-\left(\varepsilon_6\right)_{ult} < \varepsilon_6 < \varepsilon_{6ult}$$

For both cases: Each component of stress (strain) has its own failure condition (criterion) and it is not affected by the other components, i.e., **there is no interaction**

Strength differential



Fracture Criteria for Brittle Materials under Plane Stress —Maximum-Principal stress (Rankine)



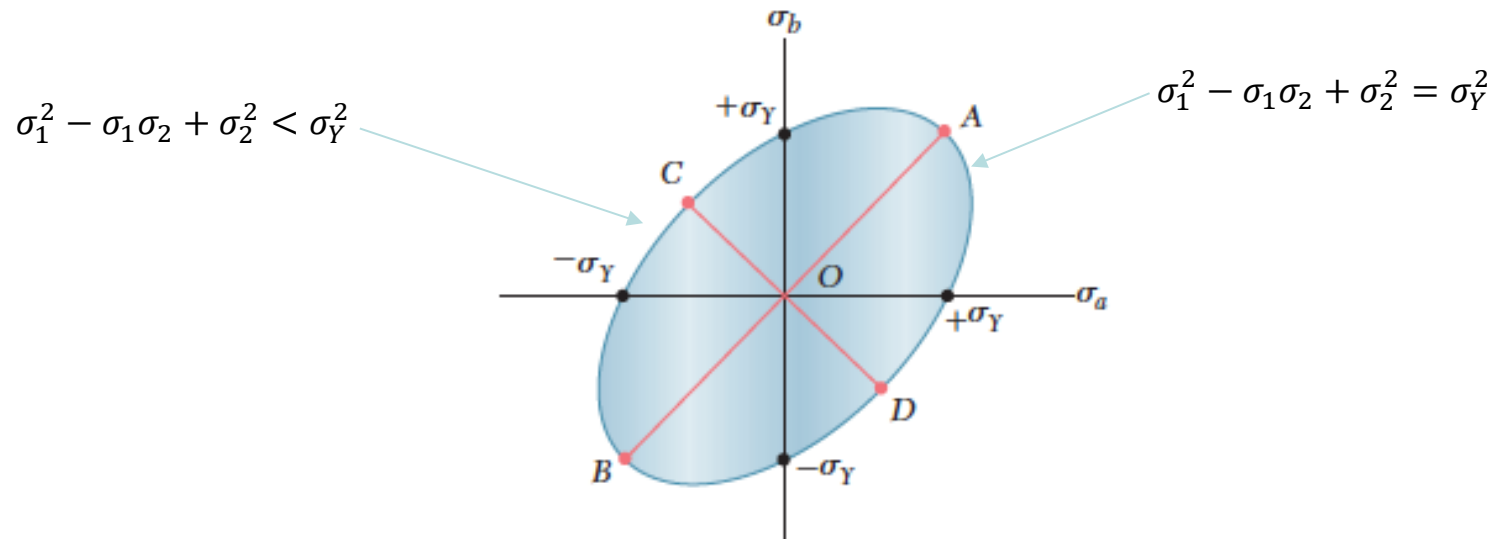
Yield Criteria for Ductile Materials under Plane Stress

Maximum Distortion Energy Criterion (Von Mises)

- For the plane stress condition, the Von Mises criterion is written:

$$\sigma_y^2 = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

Showing the mathematical equation of a rotated ellipse on the $\sigma_1 - \sigma_2$ plane.

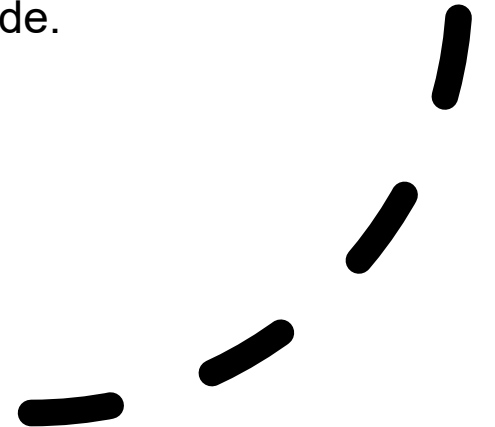


Source: after Austrian Engineer, Richard Von Mises (1883-1953)



Strength of (composite) materials

- **For composite materials:**
 - The one-constant approach for strength or for stiffness is no longer adequate
 - (For the simplest case of the unidirectional laminate) Four elastic constants are needed for the stiffness and six constants for the strength.
 - However, unidirectional composites have highly directionally dependent strengths. Therefore, for any state of applied stress, **all stress components must be examined** before judgment on the cause of failure can be made.



Failure criteria: Assumptions...



The determination of the “strength” and the behavior of an orthotropic medium is based on the assumptions that the material is:

Homogeneous

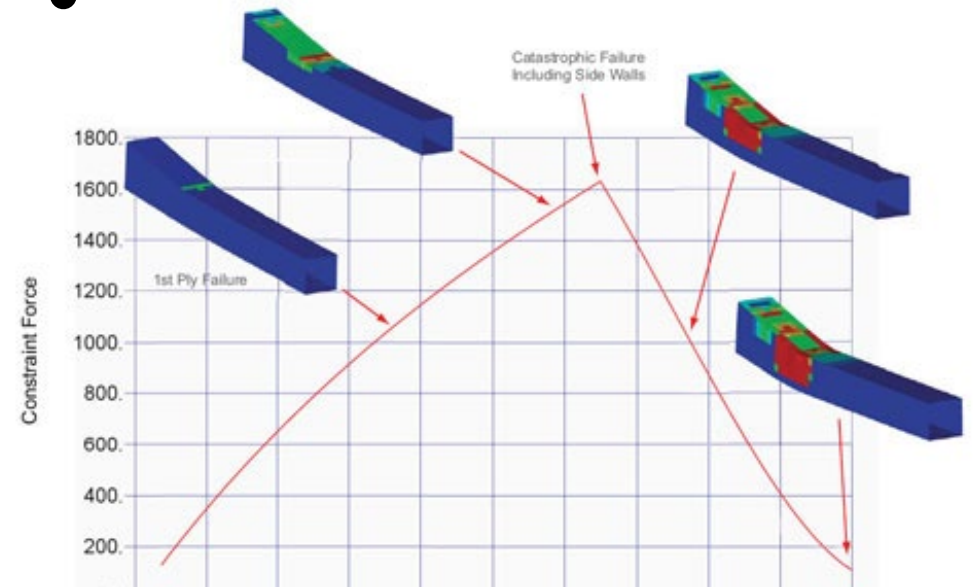
Linear elastic up to failure



For the application of the failure criteria the **strength of the material** under “uniaxial” loading should be determined experimentally.

Failure criteria

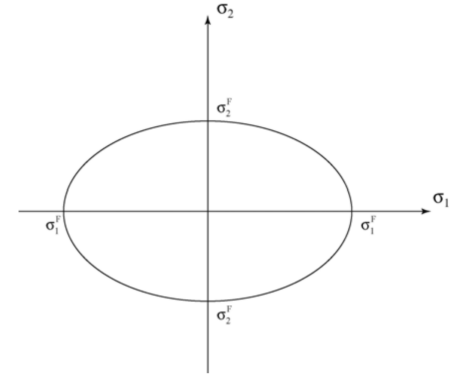
- For the analysis of composite laminates (consisting of several unidirectional plies) **we need a failure criterion for the unidirectional layer**. We would expect successive ply failures as the applied load to a laminate increases.
- The **First Ply Failure (FPF)** is followed by other ply failures until the **Last Ply Failure (LPF)** which would be the ultimate failure of the laminate.



Quadratic interaction criteria



$$F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$$



And for the strain components:

$$G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1$$



Where the F 's and G 's are strength/failure strain parameters analogous to the constants in equations of maximum stress/strain criteria. Failure occurs when equation is met (failure locus).

Use of failure criteria

- Failure criteria constitute the analytical mathematical expressions for the calculation of the failure under complex loading conditions as a function of the “basic” material properties, that can be determined through “simple” tests.
- Failure criteria serve important functions in the design and sizing of composite laminates. **However, the criteria are not intended to explain the mechanics of failure (although...).** The framework should remain the same for different definitions of failure, such as the ultimate strength, the yield point the endurance limit or even a working stress based on reliability considerations.
- **The quadratic criterion will be analyzed here.** It is simple and versatile. It considers interaction between the stress or strain components analogous to the von Mises criterion for isotropic materials.

Tsai-Hill

Criterion that has been proposed by R. Hill* for orthotropic media. It is a generalization of the von-Mises distortional energy yield criterion for isotropic materials.

$$F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L\sigma_4^2 + 2M\sigma_5^2 + 2N\sigma_6^2 = 1$$

With:

$$2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}, \quad 2H = \frac{1}{Y^2} + \frac{1}{X^2} - \frac{1}{Z^2}, \quad 2G = \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2}$$

$$2L = \frac{1}{R^2}, \quad 2M = \frac{1}{T^2}, \quad 2N = \frac{1}{S^2}$$

X, Y, Z, R, T, S: the material strength under normal and shear loads

Tsai-Hill criterion does not consider for the different strength of the anisotropic material under tensile or compressive loads or positive-negative shear...

*R. Hill, "A Theory of the Yielding and Plastic Flow of Anisotropic Metals" Proc. Royal Soc. A193, pp.281-297 (1948)

Failure tensor polynomial

For orthotropic media, in the local system:

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 + 2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

There exist 12 independent failure components that should be determined through tests

If the stresses are known for another co-ordination system then:

$$F'_{11}\sigma_1'^2 + F'_{22}\sigma_2'^2 + F'_{33}\sigma_3'^2 + F'_{44}\sigma_4'^2 + F'_{55}\sigma_5'^2 + F'_{66}\sigma_6'^2 + 2(F'_{12}\sigma_1'\sigma_2' + F'_{13}\sigma_1'\sigma_3' + F'_{23}\sigma_2'\sigma_3') + F'_1\sigma_1' + F'_2\sigma_2' + F'_3\sigma_3' - 1 = 0$$

F'_{ij} unknown, but functions of F_{ij} !

It is always preferable to calculate the stresses for the local system, where the failure components are known.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_1' \\ \sigma_2' \\ \sigma_6' \end{Bmatrix} \longrightarrow F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 + 2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

$$m = \cos\theta, n = \sin\theta$$

Determining the failure components

$$\begin{aligned} & (F_{11})\sigma_1^2 + (F_{22})\sigma_2^2 + (F_{33})\sigma_3^2 + (F_{44})\sigma_4^2 + (F_{55})\sigma_5^2 + (F_{66})\sigma_6^2 + \\ & 2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + (F_1)\sigma_1 + (F_2)\sigma_2 + (F_3)\sigma_3 - 1 = 0 \end{aligned}$$

Tensile test in 1-direction

$$\sigma_1 \neq 0, \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = \sigma_6 = 0$$

→ $F_{11}\sigma_1^2 + F_1\sigma_1 = 1$ At failure $\sigma_1 = X$

$$F_{11}X^2 + F_1X = 1$$

Compression in 1-direction

$F_{11}\sigma_1^2 + F_1\sigma_1 = 1$ At failure $\sigma_1 = -X'$

$$F_{11}X'^2 - F_1X' = 1$$

$$F_{11} = \frac{1}{XX'}, \quad F_1 = \frac{1}{X} - \frac{1}{X'}$$

accordingly:

$$F_{22} = \frac{1}{YY'}, \quad F_2 = \frac{1}{Y} - \frac{1}{Y'}, \quad F_{33} = \frac{1}{ZZ'}, \quad F_3 = \frac{1}{Z} - \frac{1}{Z'}$$

Pure shear test on 1-2 plane

For the orthotropic medium the shear strength is not dependent on the sign:

$$R^+ = R^-, \quad T^+ = T^-, \quad S^+ = S^- \longrightarrow F_{44} = \frac{1}{R^2}, \quad F_{55} = \frac{1}{T^2}, \quad F_{66} = \frac{1}{S^2}$$

$$\begin{aligned} & (F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 + \\ & 2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + (F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3) - 1 = 0 \end{aligned} \quad F_{ij}, i \neq j: \text{stress interaction terms}$$

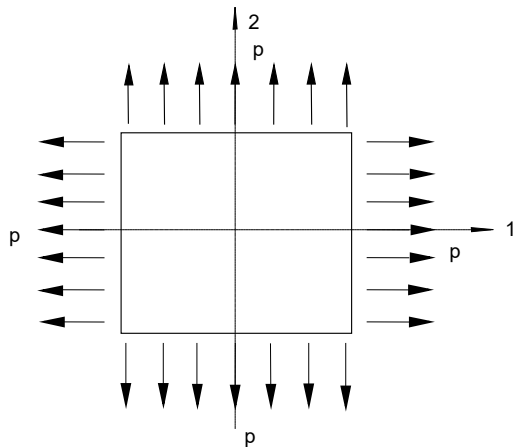
With 6 simple tests and 3 “almost” simple the failure components can be measured experimentally.

What can be done with the complex terms, that designate the stress interaction?

Determination of the stress interaction terms leads to different criteria!!!

A. Tsai-Wu¹

Determine F_{ij} through biaxial tests:



$$(F_{11} + F_{22})p^2 + 2F_{12}p^2 + (F_1 + F_2)p - 1 = 0$$

$$F_{12} = \frac{1}{2p^2} \{ 1 - (F_{11} + F_{22})p^2 - (F_1 + F_2)p \}$$

Apparently, same F_{12} value should be measured for different loading, e.g., $\sigma_1 = p$ and $\sigma_2 = 3p$

B. Tsai-Hahn¹

Theoretical determination of F_{ij} terms through the equations:

$$F_{ij} = -\frac{1}{2}\sqrt{F_{ii}F_{jj}} \quad i, j = 1, \dots, 3 \quad i \neq j \quad \text{e.g. } F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}}$$

C. Narayanaswami-Adelman²

$$F_{ij} = 0 \quad i, j = 1, \dots, 3 \quad i \neq j$$

F_{ij} should be neglected because of their low values. Good for in-plane off-axis uniaxial loading. Problems occur when complex plane stress field exists or for 3D stress fields.

D. Wu-Stachurski¹

$$F_{ij} = \frac{-F_{ii}F_{jj}}{F_{ii} + F_{jj}} \quad i, j = 1, \dots, 3 \quad i \neq j$$

Empirical, based on experimental data on specific material.

E. Hoffman³-EPFS⁴

$$F_{ij} = \frac{1}{2}(F_{kk} - F_{ii} - F_{jj}) \quad i, j, k = 1, \dots, 3 \quad i \neq j \neq k$$



Other failure criteria

- Independent conditions and interactive criteria can be combined to allow different formulations for different failure modes. These types of criteria are sometimes referred to as partly interactive criteria.
 - Simple Puck criterion
 - Modified Puck criterion
 - Hashin criterion
 - ...

Simplified Puck criterion

- Failure due to longitudinal loads and the matrix failure mode due to transverse and shear loads are considered separately.
 - The criticality of the stress state in the fiber direction is predicted with the maximum stress condition:
 - Matrix failure is predicted using the expression:
 - Tensile or compressive failure stresses are used depending on the stress state:
 - Failure is predicted to occur when either f_f or f_m reaches the value one. Hence the criterion function is defined as:

$$f_f = \left| \frac{\sigma_1}{X} \right|$$

$$f_m = \left(\frac{\sigma_2}{Y} \right)^2 + \left(\frac{\tau_{12}}{S} \right)^2$$

$$\sigma_1 \geq 0 \Rightarrow X = X_t; \sigma_1 < 0 \Rightarrow X = X_c$$

$$\sigma_2 \geq 0 \Rightarrow Y = Y_t; \sigma_2 < 0 \Rightarrow Y = Y_c$$

$$f = \max(f_f, f_m)$$

Modified Puck criterion

- Identical to the simplified Puck's criterion except for the condition for matrix failure:

$$f_m = \frac{\sigma_2^2}{Y_t Y_c} + \frac{\tau_{12}^2}{S^2} + \left(\frac{1}{Y_t} - \frac{1}{Y_c} \right) \sigma_2$$

Hashin criterion

- Fiber failure:

$$f_f = \left(\frac{\sigma_1}{X_t}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2, \quad \sigma_1 \geq 0 \quad f_f = -\frac{\sigma_1}{X_c}, \quad \sigma_1 < 0$$

- In the case of tensile transverse stress the expression for predicting matrix failure is:

$$f_m = \left(\frac{\sigma_2}{Y_t}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2, \quad \sigma_2 \geq 0$$

- A more complicated expression is used when the transverse stress is compressive:

$$f_m = \left(\frac{\sigma_2}{2S}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 + \left[\left(\frac{Y_c}{2S}\right)^2 - 1\right] \frac{\sigma_2}{Y_c}, \quad \sigma_2 < 0$$

- The more critical component is used to describe the failure mode:

$$f = \max(f_f, f_m)$$

A close-up photograph of a golf club head and a white golf ball on a green field. The club head is on the left, and the ball is on the right, both resting on the grass. In the background, a red flag is visible on a pole, and the sky is blue with some clouds. The text "Which one is the best?" is overlaid in white, centered over the image.

Which one is the best?

Experimental evidence:

In certain cases, FTP provides more accurate results compared to other failure criteria

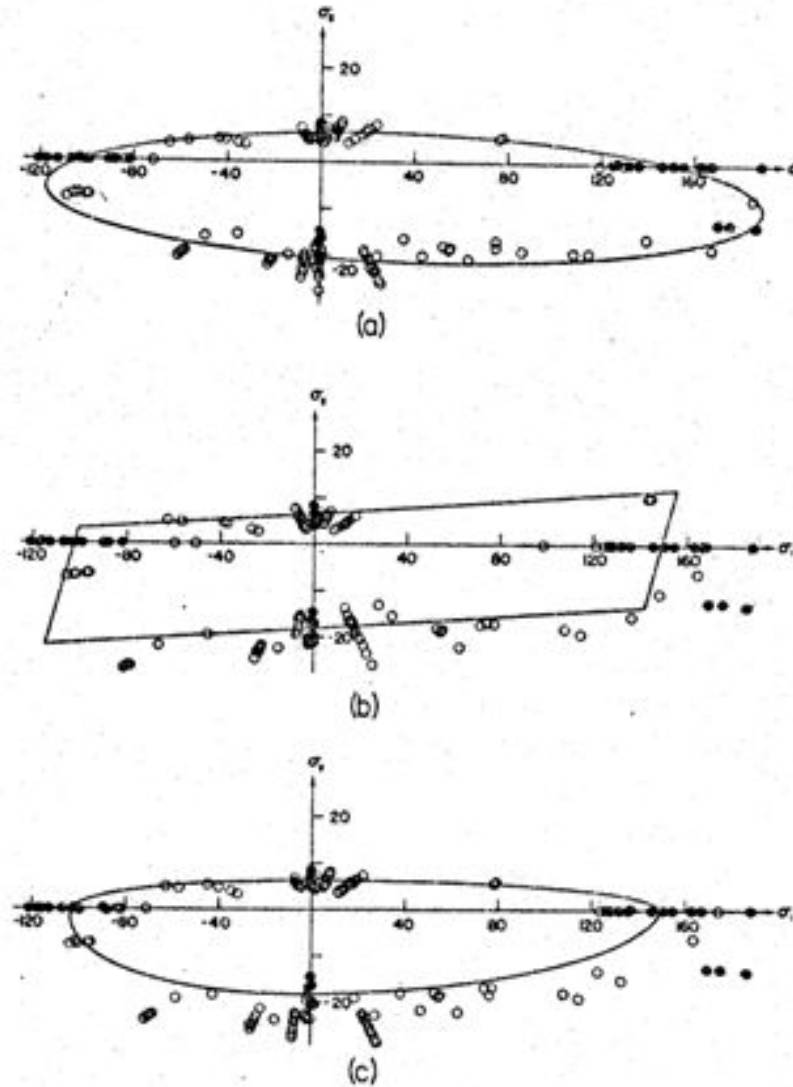


FIG. 18. Failure data convoluted onto $\sigma_1\sigma_2$ plane; stresses in [ksi]: (a) by tensor polynomial failure criterion; (b) by maximum strain failure criterion; (c) by modified von Mises-Hill failure criterion.

Use of failure criteria

- Failure prediction
- Off-axis strength prediction
- ...



Substituting, Hill's criterion can be written as: (on the local system):

$$\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\sigma_3^2}{Z^2} - \left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right) \sigma_2 \sigma_3 - \left(\frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2}\right) \sigma_1 \sigma_3 - \left(\frac{1}{Y^2} + \frac{1}{X^2} - \frac{1}{Z^2}\right) \sigma_1 \sigma_2 + \frac{\sigma_4^2}{R^2} + \frac{\sigma_5^2}{T^2} + \frac{\sigma_6^2}{S^2} = 1$$

And for plane stress (plane 1-2):

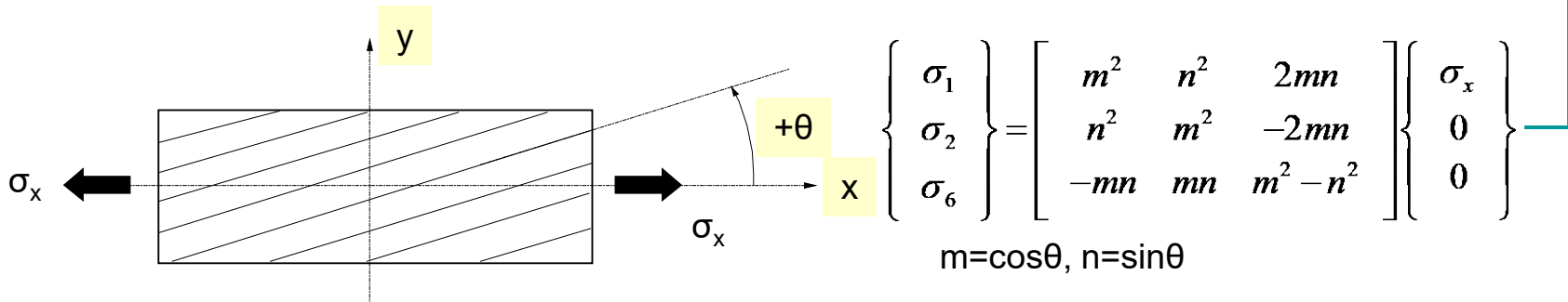
And for a transversely isotropic medium (UD):

$$\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} - \left(\frac{1}{Y^2} + \frac{1}{X^2} - \frac{1}{Z^2}\right) \sigma_1 \sigma_2 + \frac{\sigma_6^2}{S^2} = 1$$

Failure prediction

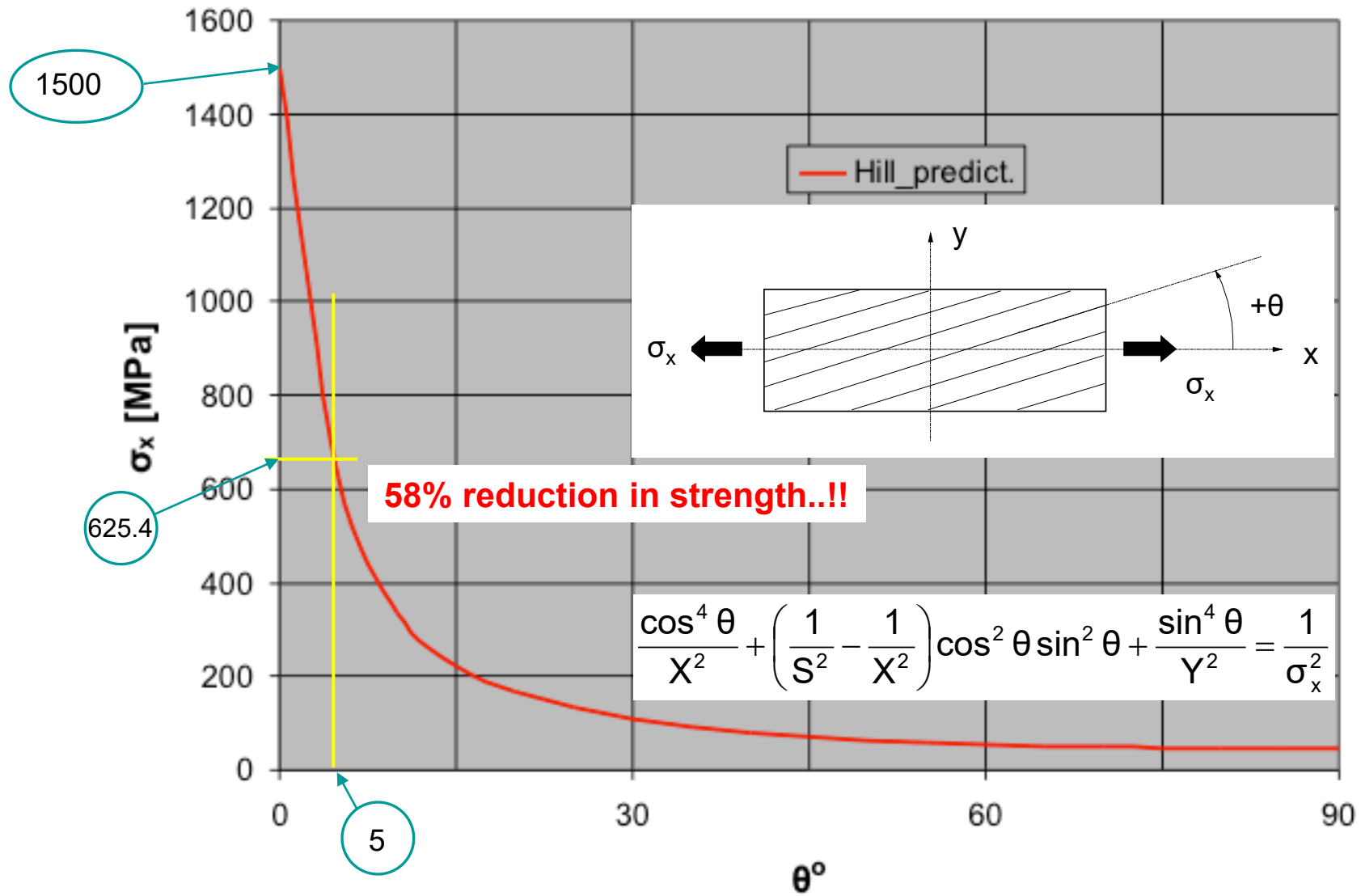
$$\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} - \frac{\sigma_1 \sigma_2}{X^2} + \frac{\sigma_6^2}{S^2} = 1$$

Example: Prediction of failure of an off-axis lamina under uniaxial tensile load (prediction of the off-axis strength!)



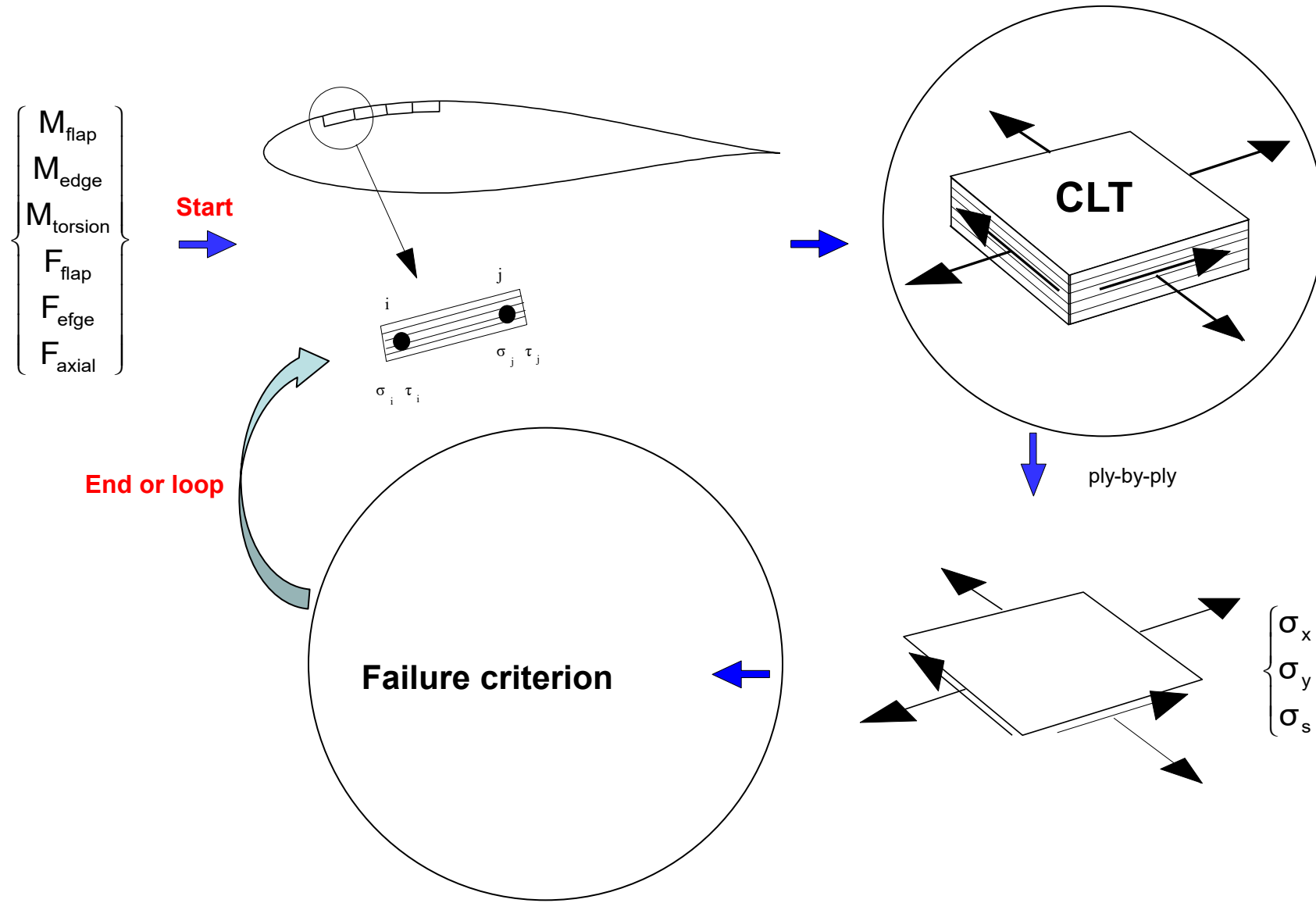
$$\frac{\cos^4 \theta}{X^2} + \left(\frac{1}{S^2} - \frac{1}{X^2}\right) \cos^2 \theta \sin^2 \theta + \frac{\sin^4 \theta}{Y^2} = \frac{1}{\sigma_x^2}$$

Off-axis strength of UD FRP



Implementation of failure criteria in a design cycle

Or - How and where do we use a failure criterion

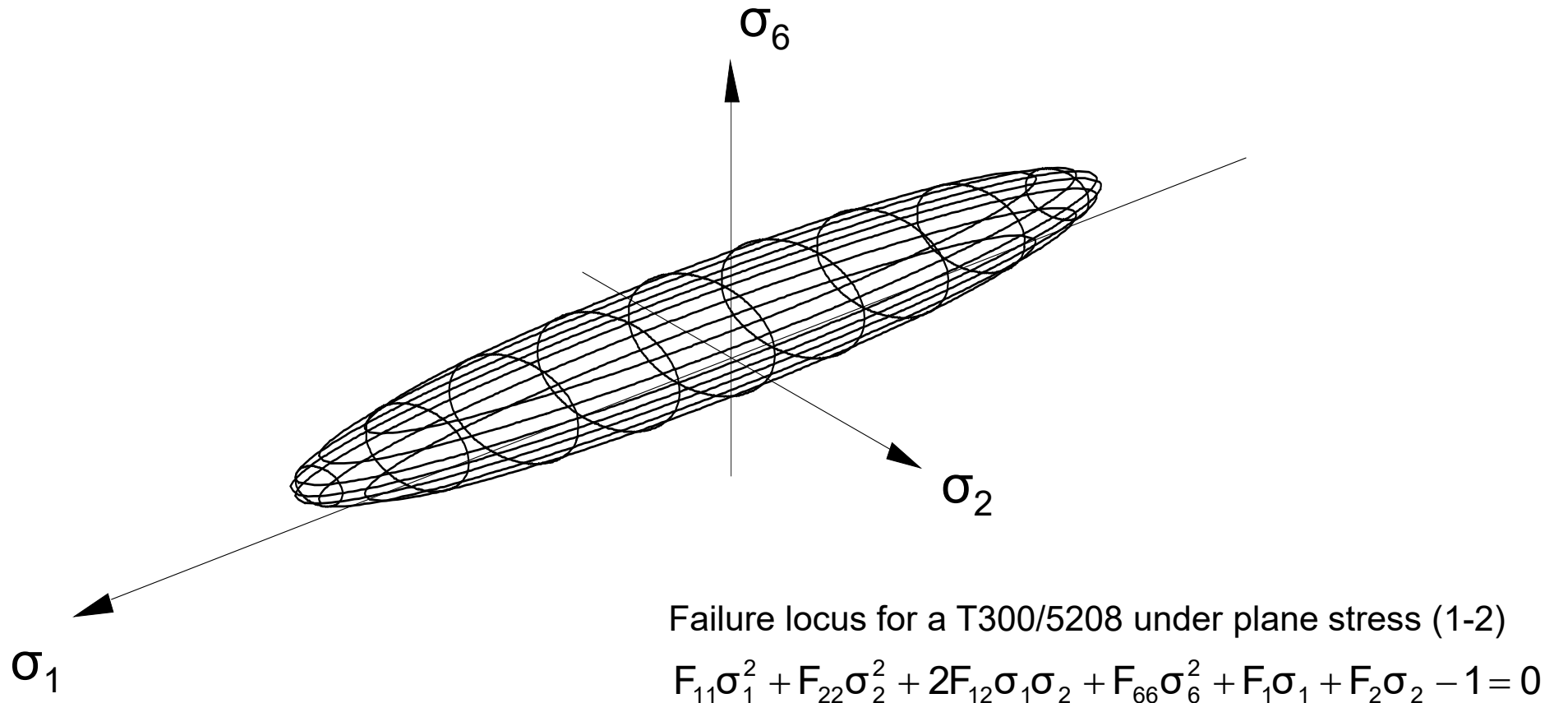


Failure Loci

Failure criteria can assist material selection procedures

A failure locus offers a tool for the evaluation of failure criteria by comparing with available experimental data.

A failure locus, especially for anisotropic media, helps the designer to understand the material capacity for transferring loads – it helps in material selection at preliminary design stages



For the transversely isotropic medium (UD) only 5 failure stresses should be determined: X, X', Y, Y', S. (the same for the orthotropic medium)

Failure stresses, in MPa, of (UD) composite materials

Commercial name	Material	X	X'	Y	Y'	S
T300/5208	Gr/Ep	1500	1500	40	246	68
B(4)/5505	B/Ep	1260	2500	61	202	67
AS/3501	Gr/Ep	1447	1447	52	206	93
Scotchply 1002 (3M)	GI/Ep	1062	610	31	118	72
Kevlar 49	Ar/Ep	1400	235	12	53	34

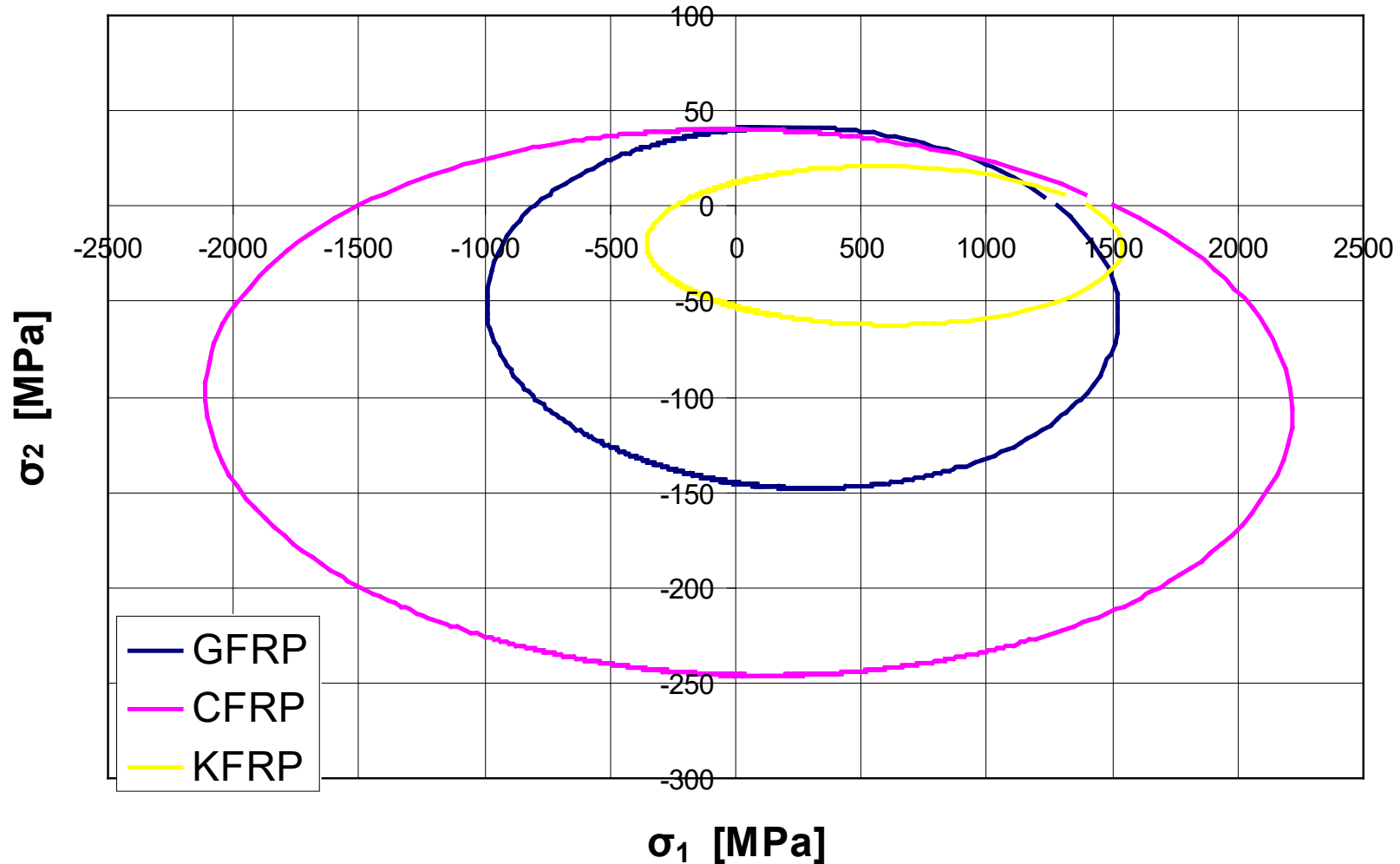
Material selection



Representative failure locii for different composite materials ($F_{12}=-F_{11}/2$).

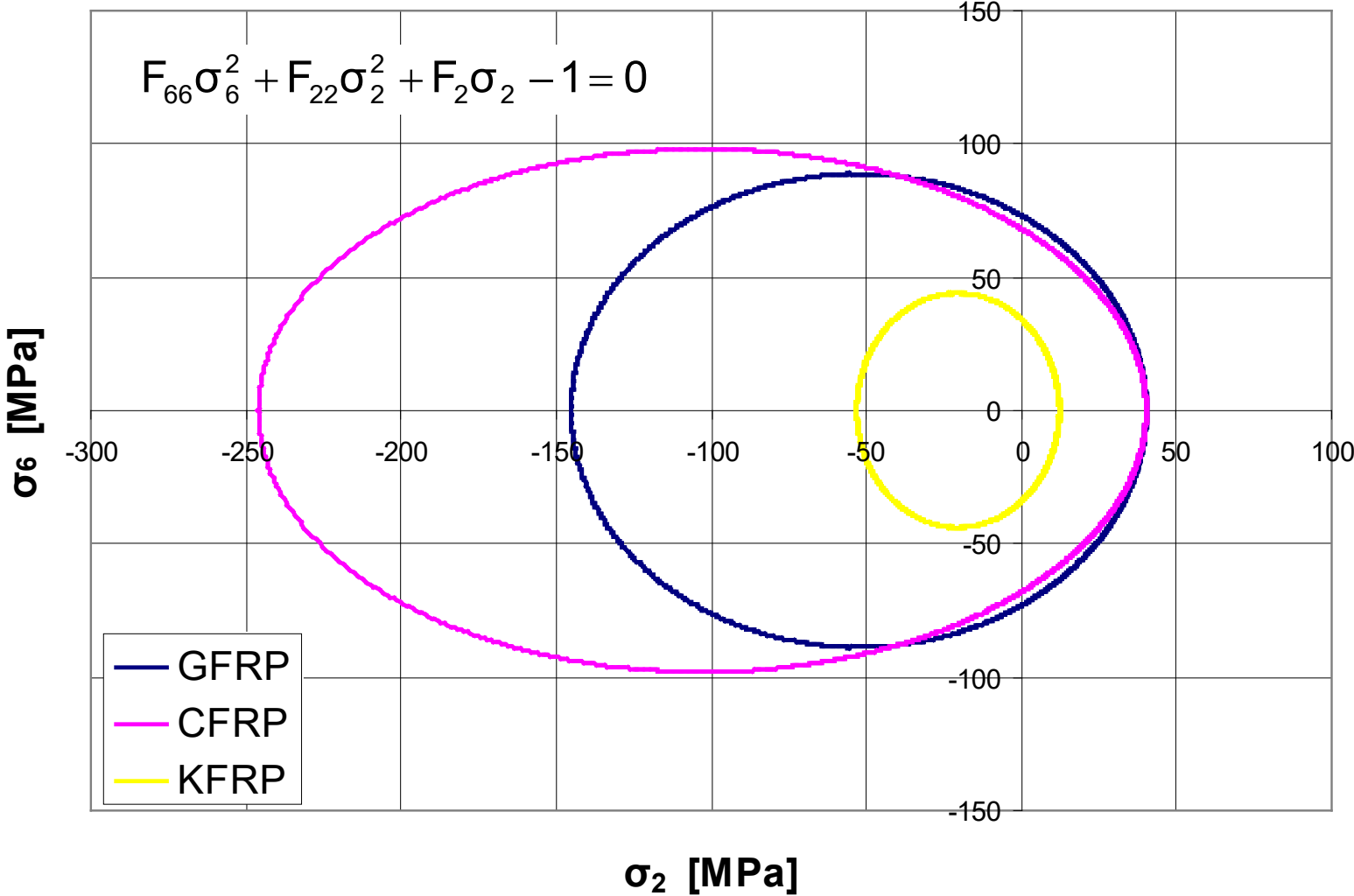
Failure locii in $(\sigma_1-\sigma_2)$

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_1\sigma_1 + F_2\sigma_2 - 1 = 0$$



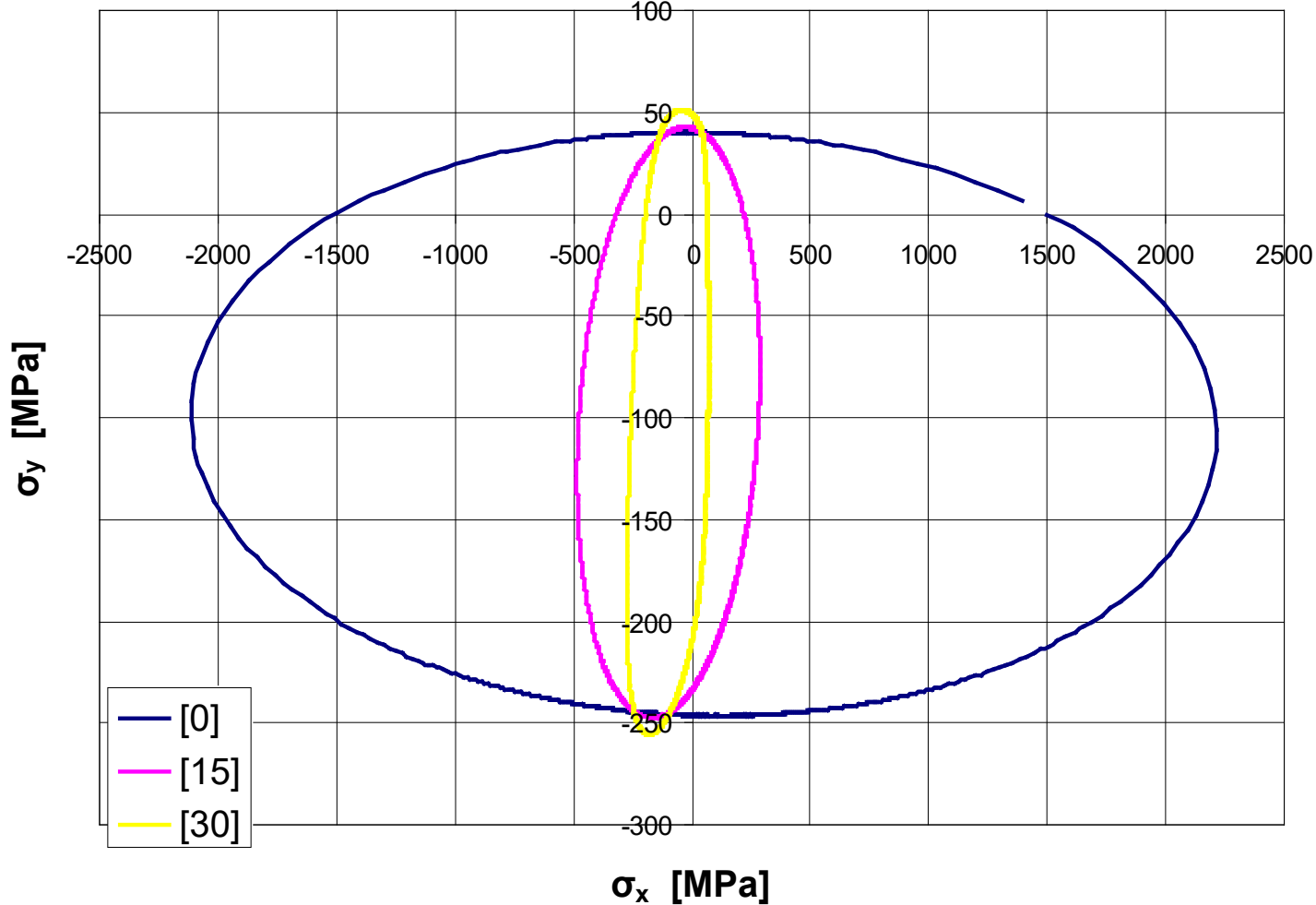
Representative failure locii for different composite materials

Failure locii in (σ_2 - σ_6) plane



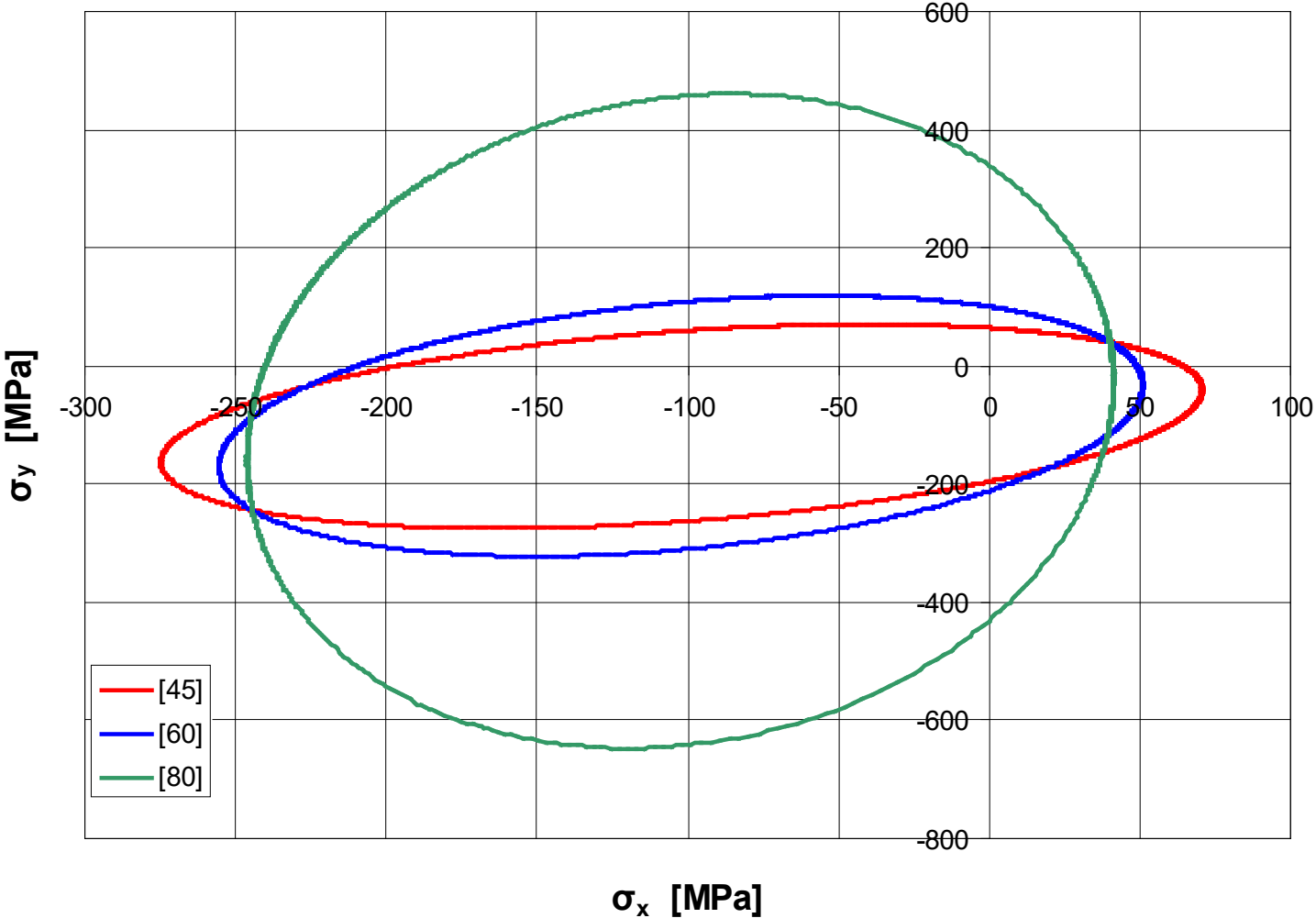
Failure locii for T300/5208 for different off-axis angles (1-2 plane)

Off-axis failure locii



Failure locii for T300/5208 for different off-axis angles (1-2 plane)

Off-axis failure locii

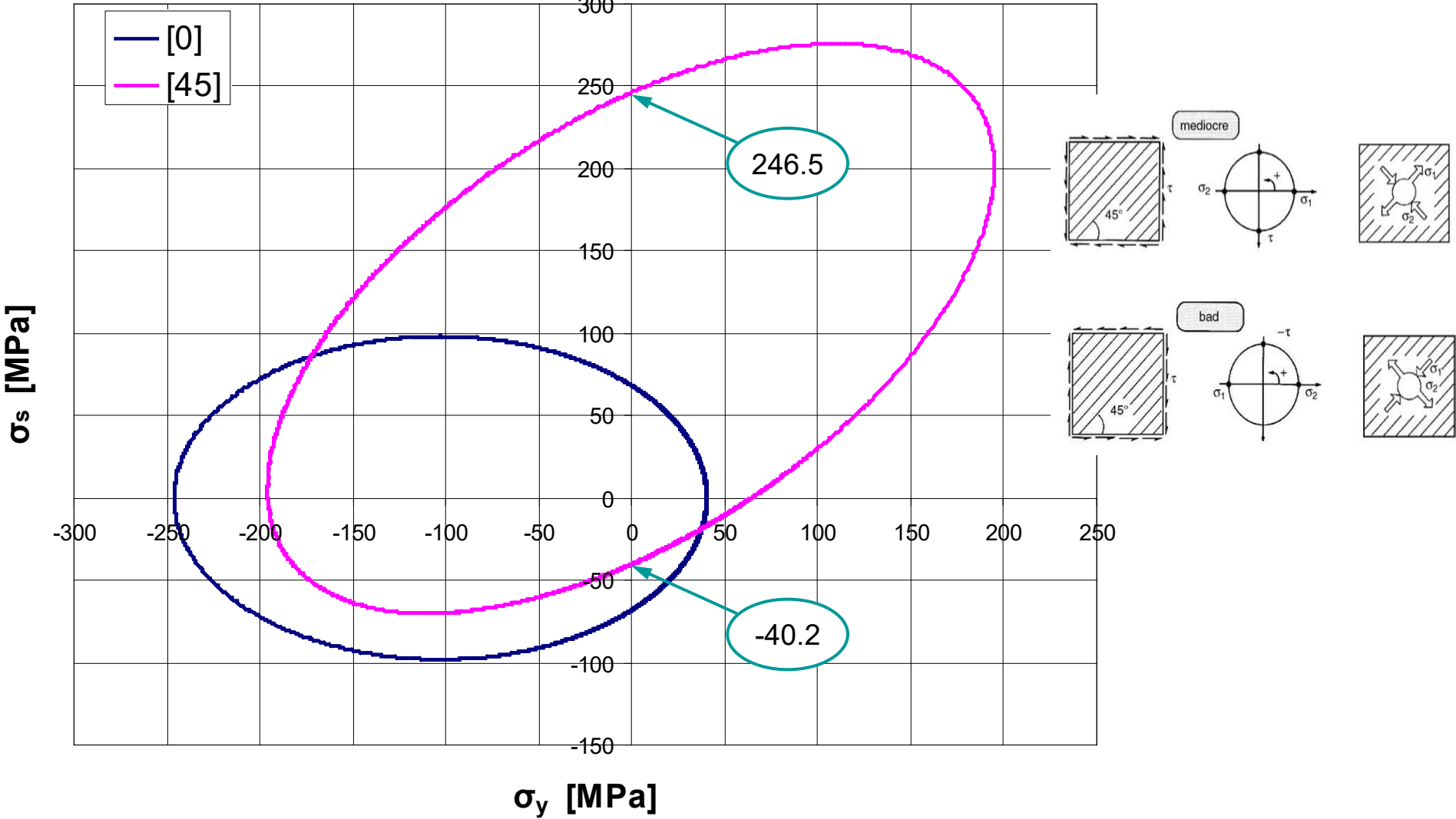


Failure locii for T300/5208 for different off-axis angles (2-6 plane)

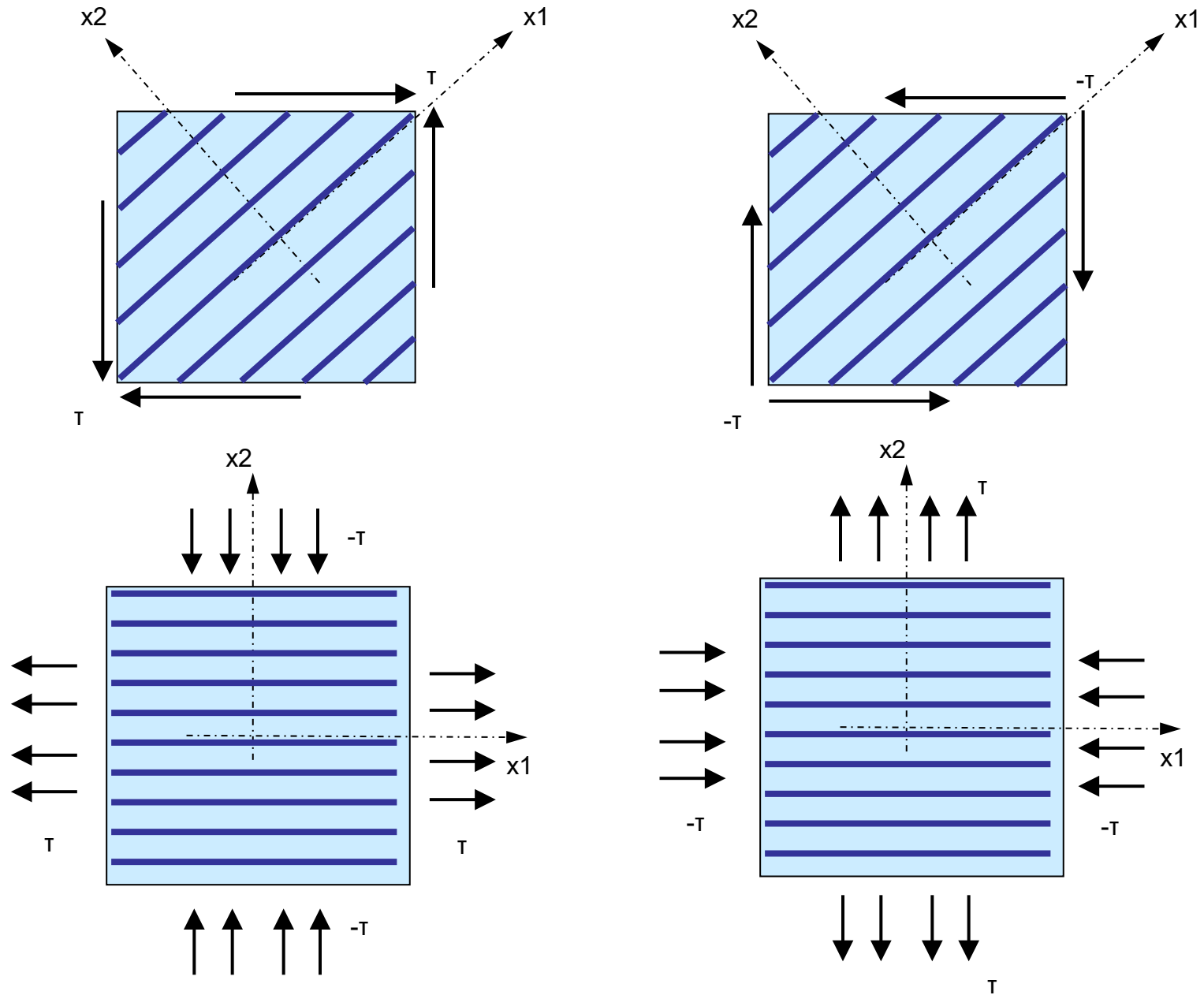
T300-N5208

Around 6 times higher..!!

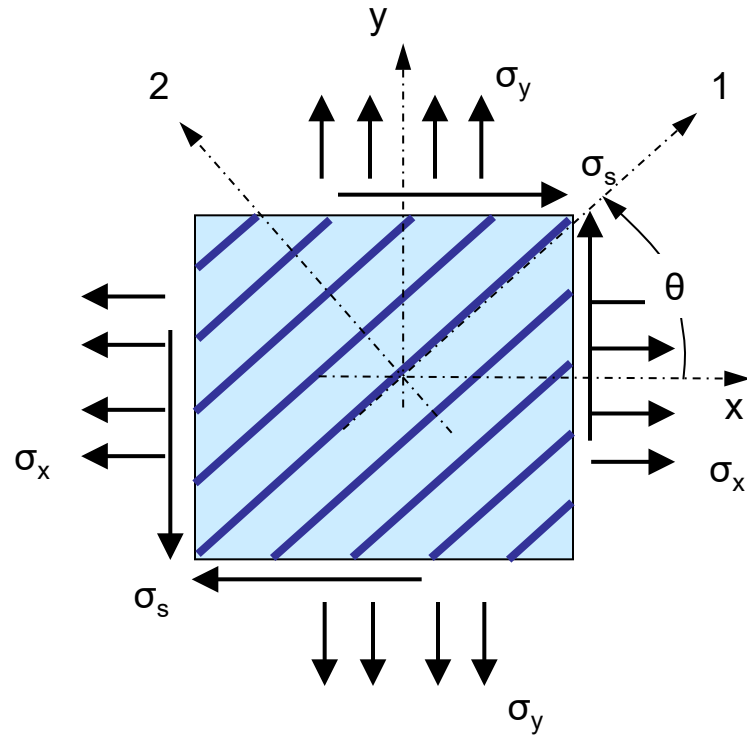
Different "positive" and "negative" shear!



Equivalent stress states for on-axis co-ordinate and 45° off-axis co-ordinate systems



Example 1: Determine the optimum fiber angle for maximum strength under a given plane stress state.



A. Principal stress orientation and value:

$$\tan 2\theta_\sigma = \frac{2\sigma_s}{\sigma_x - \sigma_y}$$

$$\sigma_{I,II} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_s^2}$$

B. Minimizing the failure tensor polynomial:

$$f(\sigma_{ij}) = F_{ij}\sigma_i\sigma_j + F_i\sigma_i - 1 \leq 0 \quad i, j = x, y, s$$

The result is dependent on the selection of the failure criterion

Consider the next loading case of a laminate made of T300/N5208:

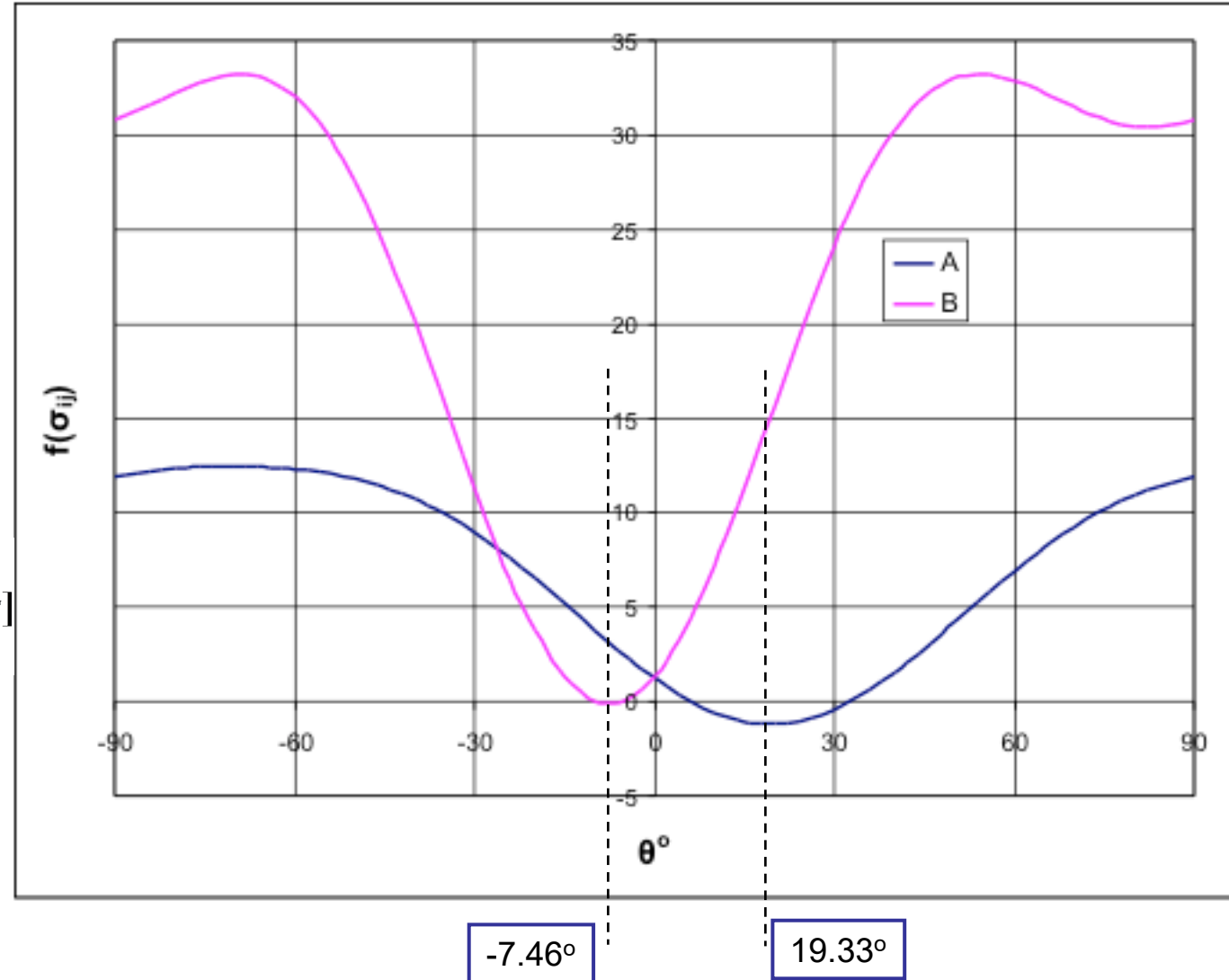
Loading case I:
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} -450 \\ -225 \\ -90 \end{Bmatrix} \text{ [MPa]} \rightarrow \begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} -193.43 \\ -481.57 \end{Bmatrix} \text{ [MPa]} \quad \theta_\sigma = -70.67^\circ$$

Fibers along an angle of 19.33°

Loading case II:
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} 450 \\ -225 \\ -90 \end{Bmatrix} \text{ [MPa]}$$

$$\begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} 461.79 \\ -236.79 \end{Bmatrix} \text{ [MPa]}$$

$\theta_\sigma = -7.46^\circ$



Coincidence for both criteria

Discussion

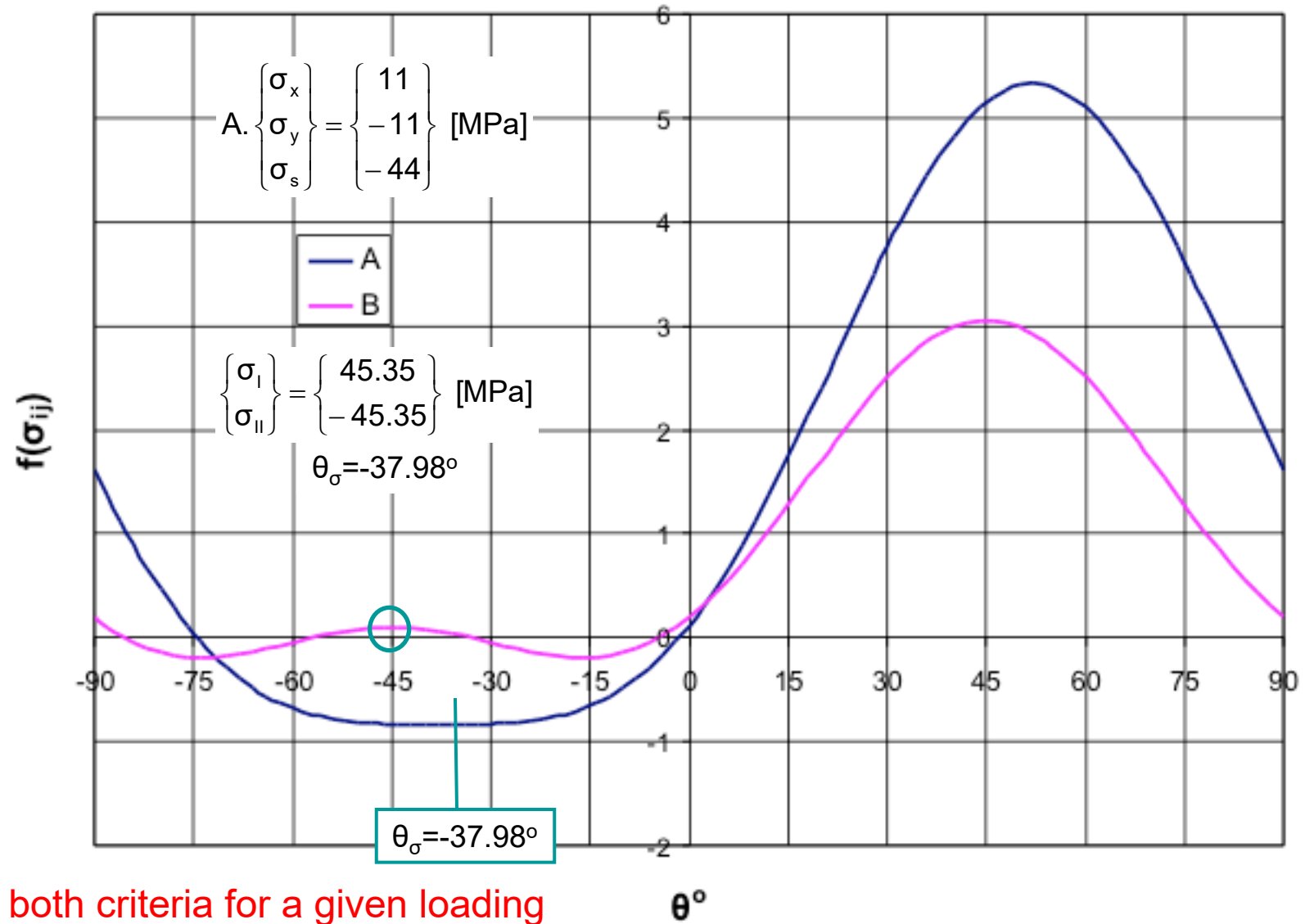
Previous results (for the given material and loading) verify the hypothesis that the fibers should be placed along the direction of the principal normal stresses.

However this is not the case for all materials and any loading case...

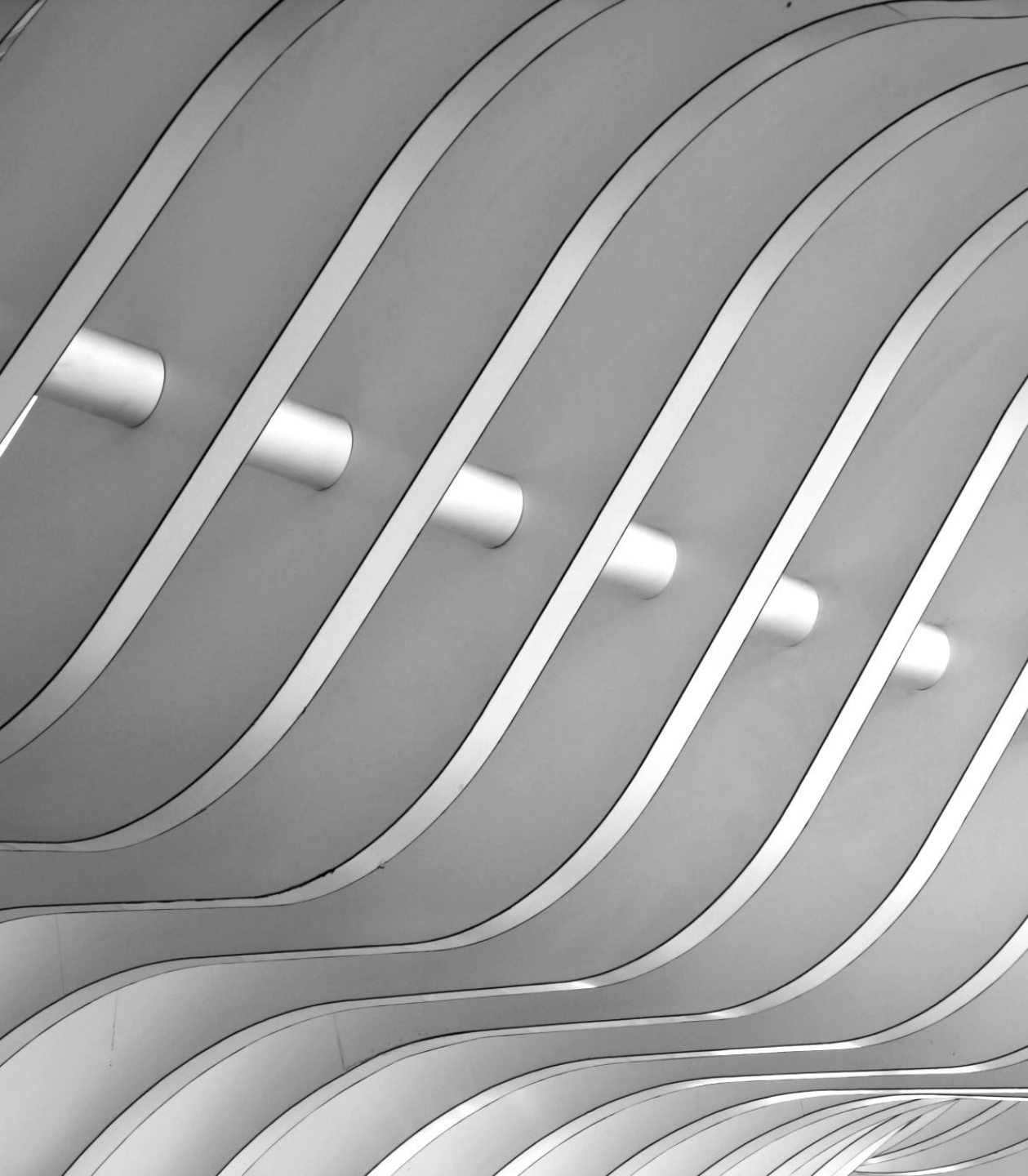
Consider the next loading cases for Kevlar 49:

$$A. \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} 11 \\ -11 \\ -44 \end{Bmatrix} [MPa] \longrightarrow \begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} 45.35 \\ -45.35 \end{Bmatrix} [MPa] \quad \theta_\sigma = -37.98^\circ$$

$$\text{B. } \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} -11 \\ -11 \\ -44 \end{Bmatrix} \text{ [MPa]} \longrightarrow \begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} 33 \\ -55 \end{Bmatrix} \text{ [MPa]} \quad \theta_\sigma = -45^\circ$$



Different estimation from both criteria for a given loading



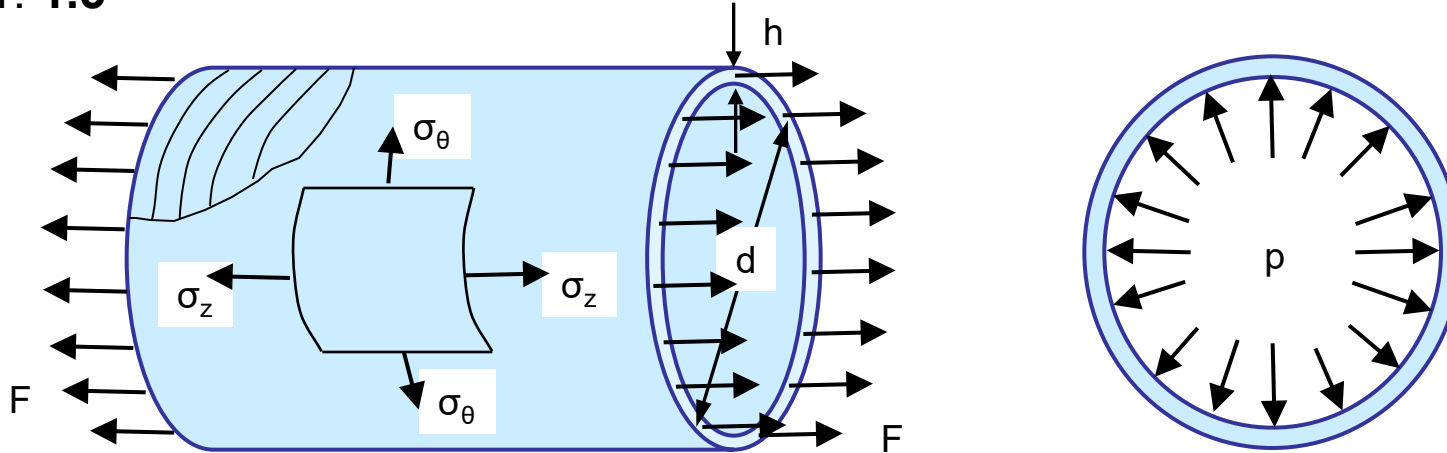
Structural design

Example II: Define the minimum thickness of the walls of a tube with a Diameter $\Phi 150$ mm that would be fabricated by the filament winding technique.

The direction of the angles (winding angle) is the transverse to the longitudinal axis of the cylinder.

Loads: Internal pressure: $p=2$ MPa, Tensile force $F=20$ kN

Safety factor: **1.5**



$$\sigma_z = \frac{F}{\pi dh}$$

$$\sigma_\theta = \frac{pd}{2h}$$

Plane stress

$$F_{11} \left(\frac{p'd}{2h} \right)^2 + F_{22} \left(\frac{F'}{\pi dh} \right)^2 + 2F_{12} \left(\frac{p'd}{2h} \right) \left(\frac{F'}{\pi dh} \right) + F_1 \left(\frac{p'd}{2h} \right) + F_2 \left(\frac{F'}{\pi dh} \right) - 1 \leq 0$$

$$F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + 2F_{12} \sigma_1 \sigma_2 + F_1 \sigma_1 + F_2 \sigma_2 - 1 = 0 \quad = 0$$

$$p' = 1.5p$$

$$F' = 1.5F$$

T300/N5208

$\gamma_F = 1.0$

$$h \geq 1.603 \cdot 10^{-3} m$$

$$h \geq 1.069 \cdot 10^{-3} m$$



9 layers of 200 μ m each



6 layers of 200 μ m each